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Stationary States in a Doubly Nonlinear Trimer Model of Optical Couplers

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Abstract

We determine analytically all stationary states of an inhomogeneous three coupler model in which two nonlinear waveguides interact with a third linear guide. The stability of these states is investigated and bifurcation points are determined.

All-optical switching devices have been a subject of intensive investigations during the past few years because of their potential applications in high-speed communication devices [1]–[9]. In this context, a number of different waveguide configurations have been studied ranging from standard two-waveguide coupler systems to arrays of coupled nonlinear optical waveguides. It has been found that three-waveguide nonlinear couplers demonstrate some distinct features in comparison with two coupled guides. In particular, three-waveguide couplers have more output states, sharper switching characteristics and are more sensitive to the input states in comparison with two-waveguide couplers. These features are of importance for applications even though improved switching characteristics are typically accompanied by undesired features, such as the increase in the required switching power.

A system of nonlinear couplers is described by the well-known discrete self-trapping equation (DST) [4, 10]. The latter was introduced in Ref. [10] as a general discrete system of equations occurring in a variety of nonlinear problems. This equation models, in particular, the so-called self-trapping phenomenon that may be considered as a rather universal mechanism of energy localization in the physics of condensed matter. Using the notation of Ref. [10] we write the DST in the form

$$iA_{jt} + \sum_k m_{jk} A_k + \gamma |A_j|^2 A_j = 0, \quad (1)$$

where A_{jt} denotes the time derivative of the amplitude in the j -th node or coupler, m_{jk} is related to the evanescence coupling of neighboring nodes and γ incorporates the Kerr-medium nonlinearity as well as the initial input power. Equation (1) can be solved analytically only in some special cases such as the dimer [1–3, 10–13], some configurations of the trimer [7, 14–16] and some other very specific configurations [17, 18]. In the case of the dimer the self-trapping transition can be studied analytically both in what regards the occurrence of a bifurcation in the stationary states of the system [10] but also in investigating its complete time dependent properties [11–13].

A model describing selftrapping in a nonlinear dimer interacting with an additional “linear” lattice site has been introduced in [19]. This model, describes in an optics context a system of two nonlinear couplers interacting with a third linear coupler. It has been found [8, 9], that such a system allows to improve switching characteristics of nonlinear directional couplers. Lower power level is required for the device to operate and more abrupt switching is possible when the additional third linear coupler is added. The basic equations that describe the dynamics of a doubly nonlinear trimer (DONT) with the third site being “linear” may be written in the following form:

$$iA_{1t} - A_2 - WA_3 + \gamma |A_1|^2 A_1 = 0, \quad (2)$$

$$iA_{2t} - A_1 - WA_3 + \gamma |A_2|^2 A_2 = 0, \quad (3)$$

$$iA_{3t} = WA_1 + WA_2, \quad (4)$$

with γ the nonlinearity parameter and W being responsible for the energy exchange between different modes (in optics W corresponds to the normalized linear coupling constant). The value of $P = |A_1|^2 + |A_2|^2 + |A_3|^2$ is a constant and it is customary to normalize it to one, i.e. $P = 1$. The system of eqs (2)–(4) represents a conservative system with the Hamiltonian $H = A_1 A_2^* + A_1^* A_2 + W(A_1^* A_3 + A_1 A_3^* + A_2^* A_3 + A_2 A_3^*) - (\gamma/2)(|A_1|^4 + |A_2|^4)$. Our primary motivation in this paper is to find stationary solutions of eqs (2)–(4) and to investigate their stability.

Consider the stationary solutions of eqs (2)–(4) of the form $A_j = \exp(i\omega t)\phi_j$. After substituting these expressions into eqs (2)–(4) we obtain the following algebraic equations for ϕ_j .

$$\omega\phi_1 + \phi_2 + W\phi_3 - \gamma |\phi_1|^2 \phi_1 = 0, \quad (5)$$

$$\omega\phi_2 + \phi_1 + W\phi_3 - \gamma |\phi_2|^2 \phi_2 = 0, \quad (6)$$

$$\omega\phi_3 + W\phi_1 + W\phi_2 = 0. \quad (7)$$

After resolving these algebraic relations we find analytic expressions for the stationary solutions of eqs (2)–(4); these results are plotted in Fig. 1. We follow the notation of [10], and use the symbols \uparrow , \downarrow and \bullet to denote the symmetric, antisymmetric and localized solutions, respectively, in the limit $\gamma \rightarrow \infty$. It can be easily checked by direct substitution that the following parametric forms provide analytical expressions for the different stationary state branches. We assume, without any loss of generality, that $W > 1$ and distinguish two cases. For $\omega > W^2$ we obtain the explicit

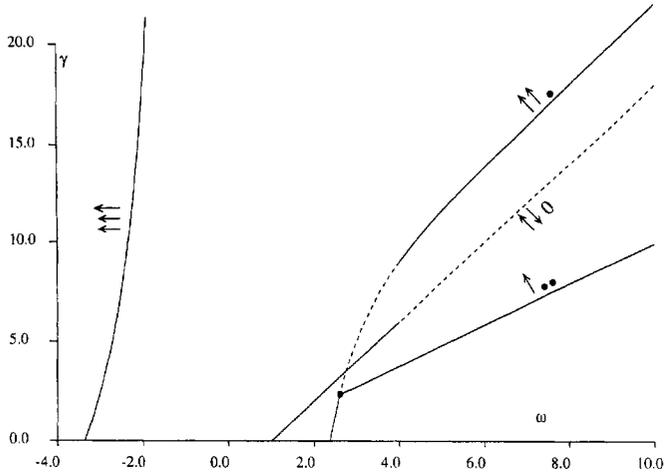


Fig. 1. Stationary solutions for DONT for different values of nonlinearity γ and frequency ω for $W = 2$. The symbols \uparrow , \downarrow and \bullet represent a symmetric, antisymmetric and selftrapped dimer state respectively in the limit $\gamma \rightarrow \infty$. Stable solutions are represented with solid lines and unstable solutions with dashed lines. A bifurcation, denoted by a bullet, occurs at $\omega_{cr} = -1 + (1 + 3W^2)^{1/2}$, or approximately at $(\omega_{cr}, \gamma_{cr}) \approx (2.4641, 2.8747)$.

expressions:

$$\phi_1 = (1 - W^2/\omega)^{1/2} \exp(-z), \quad (8)$$

$$\phi_2 = -(1 - W^2/\omega)^{1/2} \exp(+z), \quad (9)$$

$$\phi_3 = (W/\omega)(1 - W^2/\omega)^{1/2} 2 \sinh(z), \quad (10)$$

where the parameter z was introduced using the relation $2 \cosh(2z) = (\omega^2 - W^2)/(\omega - W^2)$. The low-frequency branches ($\omega < W^2$) can be expressed similarly by

$$\phi_1 = (W^2/\omega - 1)^{1/2} \exp(-z), \quad (11)$$

$$\phi_2 = (W^2/\omega - 1)^{1/2} \exp(+z), \quad (12)$$

$$\phi_3 = -(W/\omega)(W^2/\omega - 1)^{1/2} 2 \cosh(z), \quad (13)$$

with the parameter z determined now through the condition $2 \cosh(2z) = (\omega^2 - W^2)/(W^2 - \omega)$. The critical point at $\omega_{cr} = -1 + (1 + 3W^2)^{1/2}$, is a right bifurcation point. The branch emanating from this point for $\omega > \omega_{cr}$ is described analytically by $\gamma = (\omega^4 - 2\omega W^2 + W^4)/\omega^3$.

Using the analytical expressions provided through eqs (8)–(13) we can describe all solution regimes that appear in Fig. 1. The antisymmetric dimer solution with $\phi_1 = -\phi_2 = 1/\sqrt{2}$ and $\phi_3 = 0$ is given by $\gamma = 2(\omega - 1)$. This branch starts at the point $(\omega, \gamma) = (1, 0)$ in Fig. 1 and experiences a stability change at $(\omega, \gamma) = [W^2, 2(W^2 - 1)]$ equal to (4, 6) in the figure. The symmetric dimer solutions are obtained for $\phi_1 = \phi_2$. On these branches, $\phi_3 = -2W\phi_1/\omega$ and $\phi_1^2 = 1 + \omega - 2W^2\omega$, leading to $\gamma = 2(\omega^2 + 2W^2)(\omega + \omega^2 - 2W^2)/\omega^3$. These expressions give both branches labeled with two up-arrows. The change

in the stability on the right-hand branch occurs at $\omega = W^2$ as well. One may see that the self-trapped branch (labeled by two up arrows and a bullet) changes sign in ϕ_1/ϕ_2 at the point $\omega = W^2$. Finally, the state that appears as a result of the bifurcation is marked with one up-arrow and two bullets and is given by the expression $\gamma = (\omega^4 - 2\omega W^2 + W^4)/\omega^3$. The intersections of the curves with the $\gamma = 0$ axis are the eigenvalues of the linear spectral problem for the matrix m_{ij} introduced in eq. (1). The stability of the stationary states under small perturbations is an important issue for optical applications of the systems (2)–(4). We have investigated the stability of the different branches numerically. The resulting unstable regimes are presented in Fig. 1 by dashed lines in contrast to solid lines corresponding to states that seem to be stable.

In conclusion, we found the selftrapped steady state solutions for a coupler system of three-waveguides constructed as a nonlinear twin core coupler connected with a third linear core. We have investigated numerically the stability of these states and have determined the stable and unstable regimes. We note that other inhomogeneous configurations of linear and nonlinear elements, such as those studied in [9], do not lead to similar selftrapping bifurcations.

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