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Coherent propagation and energy transfer in low-dimension nonlinear arrays

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We present a theory of coherent propagation and energy or power transfer in a low-dimension array of coupled nonlinear waveguides. It is demonstrated that in the array with nonequal cores (e.g., with the central core) stable steady-state coherent multicore propagation is possible only in the nonlinear regime, with a power-controlled phase matching. The developed theory of energy or power transfer in nonlinear discrete systems is rather generic and has a range of potential applications including both high-power fiber lasers and ultrahigh-capacity optical communication systems.

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Nonlinear dynamics in discrete systems is an interdisciplinary research field that has links to a large number of areas of science and technology. A broad interest to studies of nonlinear discrete systems is based on their generic nature—a range of different physical systems can be effectively described by the same mathematical model. Nonlinear discrete systems occur in a variety of phenomena in condensed matter, nonlinear optics, biology, and other fields: from energy transport in molecular chains and protein molecules to light propagation in waveguide arrays (it is not possible to properly cite all important works in the field—see, e.g., Refs. [1–20] for particular examples relevant to the systems studied here). In this Rapid Communication we present a theory of coherent evolution and energy exchange in specific albeit generic low-dimension nonlinear discrete systems, using as a particular example a practically important application, light propagation in a multicore fiber. We demonstrate features of coherent light transmission in such multicore systems that are different from properties previously studied in the infinite nonlinear discrete lattices [1,6–16], symmetric dimers [5], and directional couplers [2,3,19,20].

The mathematical analysis of nonlinear dynamics in multicore fibers and, in a more general mathematical formulation, the nonlinear evolution of the electromagnetic field in a small number of interacting waveguides is directly relevant to the design of a new generation of fiber laser and telecommunication systems. An exponentially increasing demand for communication system capacity and the projected exhaustion of the current infrastructure ("capacity crunch" [21]) is the driving force for the introduction of spatial-division multiplexing using multicore fibers. Multicore fiber (MCF) technology enables the necessary scale-up in capacity per fiber through spatial multiplexing where individual cores serve as independent channels [22]. The important challenge here is space utilization efficiency and optimization of capacity per unit area measured in (bits/s/m^2). Interactions between the cores can be theoretically made small at the expense of space by using large core separation. However, this decreases the spatial density of capacity. More efficient space utilization is achieved in the homogeneous MCF [23] (with more dense core spacing), making positive use of the proximity of the cores to produce controlled linear core coupling. In coherent optical communication most of the linear transmission effects can be undone at the receiver by digital signal processing. However, the coupling might be affected by nonlinear effects imposing limits on enhancing performance through an increase of signal power (required to improve the signal-to-noise ratio). The nonlinearity affects energy coupling between the cores that can result in information losses. It is important, therefore, to determine the fundamental threshold for the destructive energy transfer effects.

Similar mathematical problems arise in the field of powerful fiber lasers [24,25]. The single-mode fiber can transport only the power below a certain threshold value determined by the nonlinear effects. The use of multicore fibers is a promising way for coherent combining to create high brightness sources. However, nonlinear interactions can destroy the mutual coherence. It is important, therefore, to know the limits imposed by the nonlinear interaction on the maximum power transmitted through the MCF without loss of final beam quality.

In this Rapid Communication we demonstrate that in arrays with nonequal cores (the most simple albeit general case is $N-1$ peripheral cores surrounding the central core; here N is not very large due to geometrical and manufacturing restrictions), phase matching and stable coherent propagation is possible only due to nonlinear effects for a certain power split between the cores. We solve the stability problem of steady-state propagation and derive analytical conditions of the linear instability and energy transfer. This instability is an extreme discrete limit of the classical modulation instability in the continuous media and fiber arrays [12,16,26–28].

The basic model considered here is a low-dimension version of the discrete nonlinear Schrödinger equation

$$i\frac{\partial A_k}{\partial z}+\sum_{m=0}^{N}C_{km}A_m+2\gamma_k|A_k|^2A_k=0,\quad k=0,\ldots,N. \tag{1}$$

Here A_k is a field in the kth core, with A_0 (when applied) corresponding to the central core, and $C_{km}=C_{mk}$ is the coupling coefficient between modes m and k; $C_{kk}=\beta_k$ are wave numbers in different cores that are not assumed to be the same. The phase matching and stable mutually coherent continuous-wave (cw) propagation in arrays with nonequal cores (e.g., cases 3 and 4 in Fig. 1) is provided

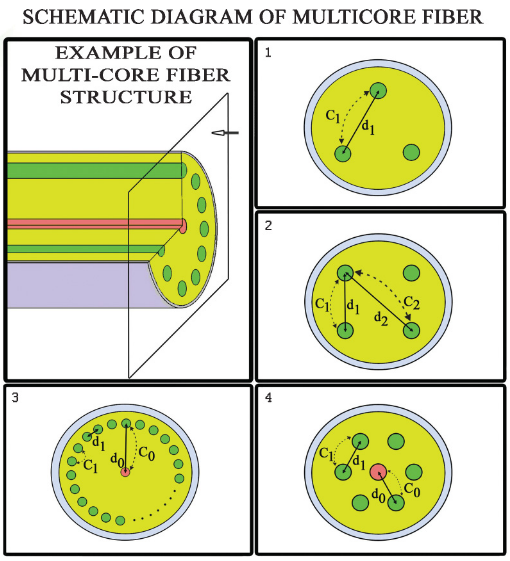

FIG. 1. (Color online) The schematic depiction of the multicore fiber.

by certain nonlinear phase shifts that we will determine below. Equation (1) governs all the designs shown in Fig. 1:

$$1: C_{mk} = C_1; \quad 2: C_{k,k+1} = C_1, \quad C_{k,k+2} = C_2;$$
$$3,4: C_{k,k\pm 1} = C_1 (k \neq 0), \quad C_{k,0} = C_0.$$

Note that in general, e.g., for systems with a distinctive central core, nonlinear coefficients in different cores might be different. Consider first the instability in cases 1 and 2 in Fig. 1. Let $A_k = (\sqrt{P_k} + a_k + i b_k) e^{iqz}$, $a_k, b_k \ll \sqrt{P_k}$, where $P_k = P_0$. Cumbersome, but direct, calculations of the dispersion relation for q show that for the case with three cores the instability occurs when $P_0 > P_{\rm th}^{(3)} = 3C_1/(4\gamma)$. In the case of four cores (case 2) the instability threshold is $P_0 > P_{\rm th}^{(4)} = (C_1 + C_2)/(2\gamma)$. When propagation constants are different, or in the case of multiple peripheral cores surrounding a central one, even the existence of a steady-state solution is nontrivial and we look at it in more detail. In the main order, dynamics in systems with similar peripheral cores can be reduced (assuming $A_k = A_1$, $k = 1, \ldots, N$) to an analysis of an effective two-core model that is a symmetric limit of multicore systems:

$$i \frac{\partial U_0}{\partial z} = -U_1 - \frac{2N\gamma_0}{\gamma_1} |U_0|^2 U_0 = \frac{\partial H}{\partial U_0^*}, \quad (2)$$

$$i \frac{\partial U_1}{\partial z} = -\kappa U_1 - U_0 - 2|U_1|^2 U_1 = \frac{\partial H}{\partial U_1^*}. \quad (3)$$

Here we introduced normalized variables:

$$A_{0,1} = \sqrt{P_{0,1}}\, U_{0,1} e^{i\beta_0 L z}, \quad z' = z/L, \quad L = \frac{1}{C_0 \sqrt{N}}, \quad (4)$$

$$P_0 = N P_1 = N^{3/2} C_0/\gamma_1, \quad \kappa = \frac{(\beta_1 - \beta_0) + 2C_1}{C_0 \sqrt{N}}. \quad (5)$$

The system of Eqs. (2) and (3) is a Hamiltonian one (as well as Eq. (1)) with the following conserved quantities: total (normalized) power P_t and the Hamiltonian H,

$$P_t = N(|U_0|^2 + |U_1|^2), \quad (6)$$

$$H = -\kappa |U_1|^2 - (U_0^* U_1 + U_1^* U_0) - |U_1|^4 - \frac{N\gamma_0}{\gamma_1} |U_0|^4. \quad (7)$$

We would like to stress that despite its simple appearance, even the stationary, steady-state solution of the Eqs. (2) and (3) is nontrivial anymore (compared, e.g., to the symmetric dimer [5]). To provide for coherent light evolution in multiple cores, the difference in propagation constants has to be compensated by the nonlinear phase shifts:

$$\{U_0, U_1\} = \{A, B\} e^{i\lambda z}, \quad \Gamma = \frac{B}{A}, \quad (8)$$

$$|A|^2 = \frac{P_t}{N(1+\Gamma^2)}, \quad \lambda = \Gamma + \frac{2\gamma_0 P_t}{\gamma_1 (1+\Gamma^2)}, \quad (9)$$

$$\Gamma^4 - \left(\kappa + \frac{2P_t}{N}\right)\Gamma^3 - \left(\kappa - \frac{2\gamma_0 P_t}{\gamma_1}\right)\Gamma - 1 = 0. \quad (10)$$

The steady-state solutions and their stability for a more general situation including gain and attenuation have been considered numerically in Ref. [29]. In a dissipative system only a numerical evaluation for some specific parameters is possible and the emphasis in Ref. [29] was on the formation of localized structures. Here we are interested mainly in energy or power transfer between the cores. The relatively simple mathematical result (8)–(10) leads to quite nontrivial physical consequences. Namely, steady-state dynamics in such a system is possible only with a certain imbalance (given by factor Γ^2) between powers propagating in different cores. The physics is rather transparent—this power split is due to a nonlinear phase shift contribution to the phase-matching condition required for coherent propagation in multiple cores. Surprisingly, there are several power distributions (between central and peripheral cores) that can provide for a coherent steady-state propagation of light. The amount of power that has to be coupled to each core for steady-state evolution given by solutions of (10) depends on four parameters: (i) N, (ii) input power $P_{\rm in}$ (or total power P_t), (iii) linear phase mismatch κ, and (iv) the ratio between the nonlinear coefficients γ_0/γ_1. To get an idea of the solution structure, consider the practically important case $P_t \gg 1$. In this case from (10) we will get four families of solutions (see Fig. 2). In $\Gamma_1 = 2P_t/N$ and $\Gamma_3 = \gamma_1/(2\gamma_0 P_t)$ most of the energy propagates in the ring or central core, correspondingly. For $\Gamma_{2,4} = \pm\sqrt{\gamma_1 N/\gamma_0}$ the ratio of energy in the ring and central core is independent of propagating power. Negative Γ means out-of-phase fields in the central and peripheral cores. Figure 3 shows an excellent applicability of the analytical results.

Consider now the stability of the steady-state solutions of (8)–(10), the analog of the modulation instability for a low-dimension discrete system. The small amplitude disturbance is taken in a standard form, $\{U_0, U_1\} = \{A + a + ib, B + c + id\} e^{i\lambda z}$, for perturbations proportional to $\exp[pz]$ the growth rate of instability is

$$p^2 + 2 = -\frac{1}{\Gamma}\left(\frac{1}{\Gamma} - 4B^2\right) - \Gamma\left(\Gamma - \frac{4N\gamma_0 A^2}{\gamma_1}\right). \quad (11)$$

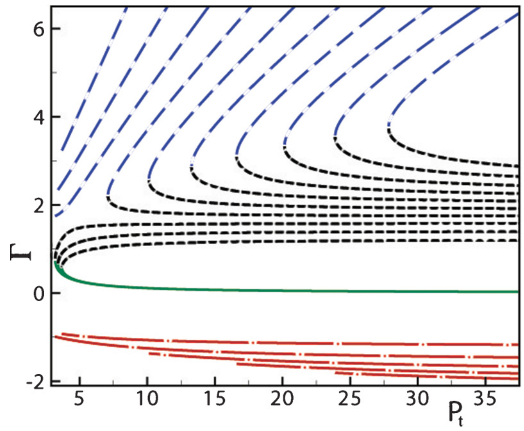

FIG. 2. (Color online) Four values of Γ corresponding to different power splits between cores as functions of total input power; here $\gamma_0/\gamma_1 = 0.5$ and $\kappa = 1$. The blue long-dashed, green solid, and red dashed-dotted branches are stable while the black short-dashed one is unstable. Here different curves for each branch correspond to N varying from 3 to 12 (from the bottom to the top). For the red short-dashed curve only odd N are shown.

In the limit $P_t \gg 1$ only mode Γ_2 is unstable. Instability results in periodic oscillations of energy between cores with an amplitude of modulations depending on total power, i.e., the relative modulation depth decreases with growing input power. The most important consequence of the instability is that it makes control of the power dynamics hardly possible. For a system with more than three cores, the instability, in general, produces stochastic modulation breaking the mutual coherence in the cores. The energy exchange oscillations can be produced not only as a result of the instability, but also as a result of initial conditions (in the case of arbitrary input powers).

The Hamiltonian structure of the equations and the additional conserved quantity greatly restricts dynamics in the considered low-dimension dynamic system, imposing constraints on the evolution of the waves and the energy exchange between cores. For instance, considering the evolution of initial powers equally distributed between all cores $|U_0|^2 = P_{\rm in}/N$, $|U_1|^2 = P_{\rm in}$, using the connections between the fields imposed by $dH/dz = 0$, it is easy to show that complete energy transfer from the outer cores to the central one is possible only for one specific value of input power (and at a specific propagation length):

$$P_{\rm in} = P_{\rm in}^{\rm th} = \frac{\kappa + 2N^{-1/2}}{\gamma_0(N+2)/\gamma_1 - 1}. \qquad (12)$$

The observed effect—localization of all initially evenly distributed power into the central core—can be considered as an ultimate discrete version of the self-focusing of light.

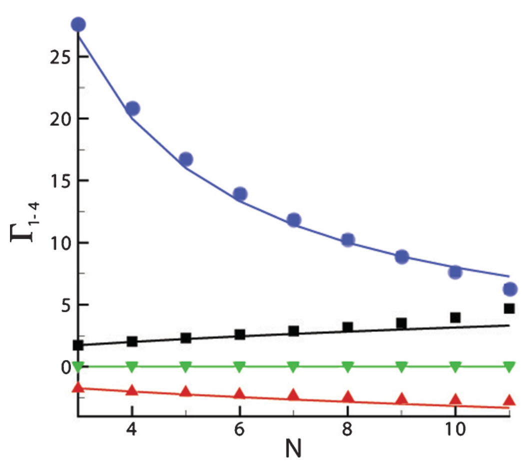

FIG. 3. (Color online) Dependence of the four solutions of Eq. (10) (shown by squares) on N. Here $P_t = 40$, $\gamma_0/\gamma_1 = 1$, and $\kappa = 1$. Solid lines are for the analytical solutions valid in the limit $P_t \gg 1$. Blue circles curve: $\Gamma_1 = 2P_t/N$; black squares line: $\Gamma_2 = \sqrt{\gamma_1 N/\gamma_0}$; green triangles line: $\Gamma_3 = \gamma_1/(2\,\gamma_0\,P_t)$; and red inverse triangle line: $\Gamma_4 = -\sqrt{\gamma_1 N/\gamma_0}$.

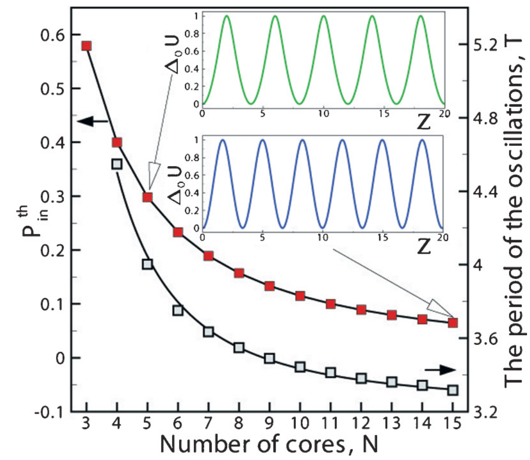

FIG. 4. (Color online) Y axis (left): The comparison of numerically calculated threshold for energy or power transfer (red markers) and the analytical formula (12) (solid line). Y axis (right): Numerically calculated period of the power oscillations (gray markers) and analytical approximation, $3.23 + 2.04/N^2$ (solid line). Insets: Energy or power transfer with distance. The complete transfer occurs only at certain distances.

Figure 4 shows a comparison of the analytical result (12) and the numerically calculated threshold of an energy transfer given by $\Delta_0 U = (N|U_0|^2 - |U_1|^2)/P_t$ $[P_t = (N+1)P_{\rm in}]$. Here $\gamma_0 = \gamma_1$, $C_0 = C_1$, $\beta_0 = \beta_1$. The period of the energy exchanges decays with N as N^{-2}.

Note that the presented theory can be easily generalized to pulse propagation and nonlinear temporal dynamics having numerous applications. In the recent important work [30] the efficiency of nonlinear matching of optical fibers through a fundamental soliton coupling from one fiber into another has been studied, opening a range of engineering applications, e.g., optimized Raman redshift and supercontinuum generation.

To conclude, in this Rapid Communication we have presented a theory of energy or power transfer in low-dimension arrays of coupled nonlinear waveguides. The developed theory is rather generic and has a range of potential applications. Without loss of generality, particular emphasis in the analysis was made on multicore fiber technology, important in the fields of both high-power fiber lasers and ultrahigh-capacity optical communication systems. We have derived for the array with nonequal cores the nonlinear phase-matching conditions that provide for stable coherent steady-state propagation in multiple cores. We solved the stability problem and found an exact analytical condition of complete energy transfer from the peripheral to the central core, the ultimate discrete analogy of the self-focusing effect.

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