Self-similar parabolic plasmonic beams

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We demonstrate that an interplay between diffraction and defocusing nonlinearity can support stable self-similar plasmonic waves with a parabolic profile. Simplicity of a parabolic shape combined with the corresponding parabolic spatial phase distribution creates opportunities for controllable manipulation of plasmons through a combined action of diffraction and nonlinearity. © 2013 Optical Society of America

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Light localization and manipulation in plasmonic systems beyond the diffraction limit offer many promising applications, including nanoantennas, biological and chemical sensing, and nanolasers [1,2]. Functionality of plasmonic devices depends on the ability to tune and control the light propagation. Recently, plasmon dispersion engineering in specifically designed waveguiding geometries was intensively discussed as a prospective tool for a passive control over the plasmon propagation, including waveform shaping for plasmon lensing and nondiffraction propagation [3–5], and plasmon nanofocusing in tapered plasmonic waveguides [6–9]. Nonlinearity (focusing or defocusing) might offer simple means for dynamic tuning of plasmonic systems and plasmonic beam propagation [10]. Nevertheless, nonlinear mechanisms of self-tuning typically employed in optical fibers are dramatically suppressed in plasmonic structures due to high losses [11,12]. However, using techniques similar to pulse manipulation in fiber-optic tapers [13], it might be possible to compensate effectively the losses in plasmonic systems [14]. In particular, tapered plasmonic waveguides were suggested for enhancing nonlinear effects and nonlinear self-focusing of plasmonic beams [15]. In this Letter we propose and explore an approach to create plasmonic beams with special shapes using tapered waveguides and nonlinear effects.

It is well established in fiber optics that the ability to manipulate signals depends critically on availability of the basic waveforms with simple shapes, e.g., square top-flat, triangular, parabolic, and so on. Moreover, focusing and defocusing nonlinearities provide power controlled possibility for generation of a variety of temporal waveforms [16–20]. In particular, an interesting class of pulses with a parabolic power distribution in the energy-containing core and parabolic (in time) phase can propagate in a fiber with normal group-velocity dispersion [16–19]. Parabolic pulses propagate in a self-similar manner holding certain relations between the varying pulse power, pulse, and chirp.

We propose a type of plasmons with self-similar profile propagating in nonlinear tapered waveguides, transferring the knowledge of the pulse manipulation from fiber optics to plasmonics. We introduce the general theory that can be used in a range of current and future plasmonic systems, and also present numerical results for a particular implementation with specific parameters.

We consider a tapered metal–dielectric–metal slot waveguide filled with a nonlinear dielectric material (see Fig. 1). We assume that the tapering angle ϕ is small, so that the adiabatic approximation remains applicable. Within this approximation, we consider that a variation of the waveguide thickness along the propagation direction is negligible, $dh/dz \ll 1$. Such an approach allows us to present the plasmonic field in the form of an eigenmode envelope slowly varying with propagation, $H = A(z,y)H_0(x,\beta)\exp(i\int\beta \mathrm{d}z)$, where $A(z,y)$ is the slowly varying complex amplitude, $H_0(x,\beta)$ is the local plasmonic eigenmode with wavevector $\beta(h)$ of slot waveguide with thickness h. Substituting this field representation into the wave equation, we derive the nonlinear Schrodinger equation with the coefficients varying along the propagation [11,15],

$$2i\sigma\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial y^2}I + iA\left(\frac{d\sigma}{dz} + \Gamma\right) + N_{\mathrm{nl}}|A|^2A = 0, \quad (1)$$

where the $I = \langle E_{x0}^2 + E_{z0}^2\rangle_x$ is the effective beam intensity, $\sigma = \langle E_{x0}H_{y0}\rangle_x$ is proportional to the overall energy flow in the propagation direction per unit length, $\Gamma = \langle \varepsilon''(E_{x0}^2 + E_{z0}^2)\rangle_x$ is the effective dissipation defined with the mode structure, where ε'' is the imaginary part of the

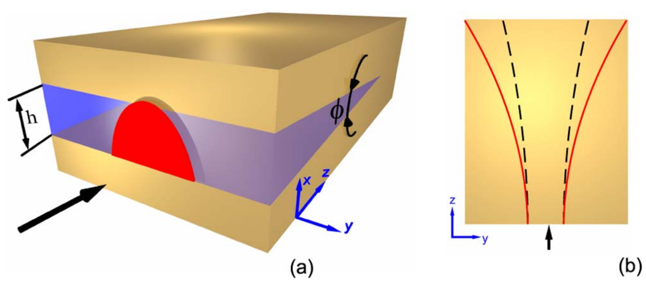

Fig. 1. (Color online) (a) Schematic of a tapered metal–dielectric–metal waveguide. (b) Schematic of the plasmonic beam propagation in linear (dashed curve) and nonlinear (solid curve) regimes.

metal permittivity, and $N_{\mathrm{nl}} = \langle \alpha(E_{x0}^2 + E_{z0}^2)^2 \rangle_x$ is the effective nonlinear coefficient, α is the third-order nonlinear coefficient. Here the symbol $\langle \cdot \rangle_x$ corresponds to the integration over the transverse coordinate x, and $E_{x,y0}$ are the electric field components of the corresponding eigenmode.

Effective losses $(d\sigma/dz + \Gamma)$ in Eq. (1) are composed of two terms: Γ describes the electromagnetic damping due to the field penetration into lossy metals, and $d\sigma/dz$ appears solely due to waveguide tapering. Quantity σ decreases with the slot width, so that the derivative $d\sigma/dz$ is negative, hence, it compensates the material losses in the taper. It is possible to show that the condition $d\sigma/dz + \Gamma = 0$ describes the taper where the effective losses in the system are completely compensated by tapering (although the total energy flow is still dissipating). We note that this condition can be fairly well modeled by a straight taper with some critical angle ϕ_c [14]. For $\phi > \phi_c$ tapering leads to an effective gain of plasmons employed for linear [7] and nonlinear [15] nanofocusing.

Next, we search for solutions of Eq. (1) in the self-similar form, $A = a(z)u(z,\xi)\exp[iC(z)y^2]$, where $\xi = y/b(z)$, and a, b, and C are functions describing the amplitude, width, and phase chirp, respectively [18]. Substituting this form of complex amplitude A into Eq. (1) and enforcing the following relations,

$$\frac{2\sigma}{a}\frac{da}{dz} + 2IC + \left(\frac{d\sigma}{dz} + \Gamma\right) = 0,$$
$$-2\sigma\frac{dC}{dz} - 4IC^2 - \frac{Na^2}{b^2} = 0,$$
$$-2\frac{\sigma}{b}\frac{db}{dz} + 4IC = 0, \qquad (2)$$

we finally derive a partial differential equation describing the evolution of parabolic plasmonic beams,

$$2i\sigma\frac{\partial u}{\partial z} + \frac{I}{b^2}\frac{\partial^2 u}{\partial \xi^2} + N_{\mathrm{nl}}a^2(\xi^2 + |u|^2)u = 0. \qquad (3)$$

Equation (3) shows that the plasmonic beam with a parabolic profile $A(z=0) = a(0)[(1 - x^2/b(0)^2)]^{1/2}$ preserves its parabolic nature under the constrains Eq. (2), so that its amplitude u remains self-similar with propagation, $|u|^2 = (1 - \xi^2)$.

We illustrate the general theory by solving Eq. (1) by the beam propagation method for realistic parameters. For our simulations, we consider a nonlinear dielectric with permittivity $\varepsilon_d = 4.84$, sandwiched between two silver plates. As an example, we study the case of defocusing nonlinearity with the nonlinear coefficient is $\chi = -1.4 \times 10^{-19}\ \mathrm{m}^2/\mathrm{V}^2$, and the light wavelength of 1550 nm [10]. We also consider a taper with the width of 600 nm at $z = 0$ decreasing toward the tip (note that in this case $\phi_c = 1.2°$). We select a parabolic initial beam profile, $A(z=0) = a(0)[(1 - x^2/b(0)^2)]^{1/2}$, where $b(0) = 7.5\ \mu\mathrm{m}$, $a(0)$ is the initial beam amplitude; also initial phase chirp $C(0) = 0$.

First, we study the plasmon propagation with a small initial amplitude $H_y(z=0) = 0.77\ \mathrm{A}/\mu\mathrm{m}$, in the linear regime for both untapered ($\phi = 0$) and tapered ($\phi = 1.6°$) waveguides. We observe that the self-similar parabolic nature of the beams is preserved, which is evident from the beam cross-sections after the propagation for 20 μm plotted in Figs. 2(a) and 2(b). Clearly, the numerical simulations match well the analytical predictions. We note that the field distortion and slight deviation from the parabolic shape after some propagation [see Figs. 2(a) and 2(b)] arise from the evolution of the discontinuities at the boundaries of the initial parabolic profile.

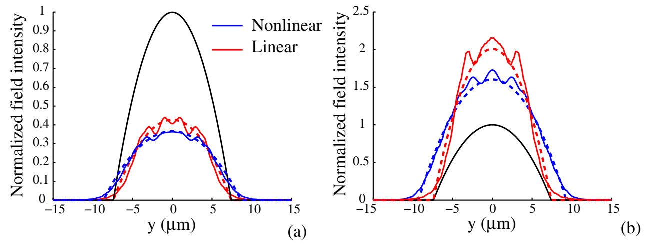

Fig. 2. (Color online) Beam cross-sections for (a) untapered and (b) tapered waveguides, respectively, after 20 μm of propagation. Solid curves correspond to numerical results, whereas dashed are our analytic predictions. Solid parabolic curve corresponds to the input beam profile.

We observe that in a slot waveguide the amplitude decreases with propagation, in contrast to the amplitude growth in the case of tapering, see Figs. 2(a) and 2(b). We expect that the amplitude growth in the case of nanofocusing in a tapered waveguide will enhance nonlinear response. In Fig. 4(a), we plot the corresponding evolutions of the analytically obtained amplitude.

Next we study the plasmon propagation in the nonlinear regime, taking the larger input amplitude $H_y(z=0) = 7.7\ \mathrm{A}/\mu\mathrm{m}$, see Figs. 3(a) and 3(b). In this case, the defocusing nonlinearity contributes to the beam diffraction enhancing the beam broadening with propagation. However, in the untapered waveguide, due to a rapid amplitude decrease caused by losses in the system, the nonlinear response decreases with the plasmon propagation, and the beam propagation does not deviate significantly form that observed in the linear regime. The latter clearly follows from the evolution of the beam cross-sections [see Fig. 2(a)] and the amplitude evolution [see Fig. 4(a)].

In the case of tapering, the amplitude grows with propagation resulting in the enhancement of the beam defocusing, which manifests itself in a stronger beam broadening with propagation, as shown in Figs. 2(b) and 3(b). From the analysis of the corresponding beam

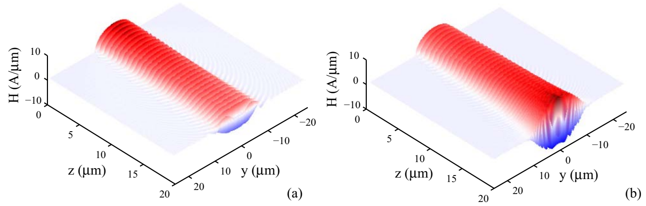

Fig. 3. (Color online) Magnetic field profiles at the metal–dielectric interface for (a) nonlinear untapered ($\phi = 0°$) and (b) nonlinear tapered ($\phi = 1.6°$) waveguides. Field profiles are plotted up-till few microns from the taper tip.

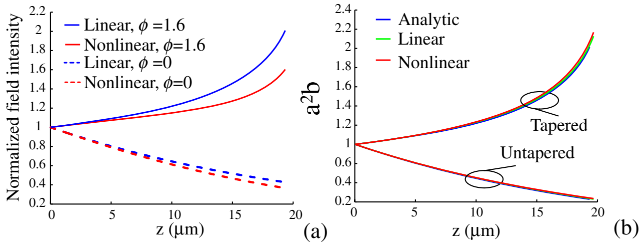

Fig. 4. (Color online) (a) Evolution of the beam amplitude with propagation obtained from the analytical theory for all studied cases. The fields are normalized to the initial amplitude $a(0)$. (b) Evolution of product a^2b with propagation for tapered and untapered waveguides, for different input amplitudes.

cross-sections, we see that in the case of a nonlinear taper both the beam width and amplitude are increased significantly in comparison with the initial beam profile. We notice that because of a strong beam broadening, in the nonlinear case the corresponding amplitude growth is weaker, see Fig. 4(a).

Finally, in order to prove the self-similar nature of the plasmonic beams, we study the evolution of the product a^2b which should remain invariant to variations of the input power. Figure 4(b) shows the evolution of the product a^2b for both tapered and untapered waveguides. Clearly, this value remains independent of the initial beam amplitude for both cases of tapering, which underlines the self-similar nature of the observed beams. We would like to stress that though in the considered numerical examples the manifestation of nonlinear effects are limited by the propagation distance (or taper angle) and chosen nonlinear parameters, the proposed plasmonic beam shaping approach is more general and can be used in low power nonlinear plasmonic structures [21].

In conclusion, we have predicted a type of self-similar plasmonic beam with a parabolic profile. Such *parabolic plasmons* with the corresponding parabolic spatial phase can be useful for controllable manipulation and processing of plasmonic waves and new design possibilities for nanophotonic devices.

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