Vector dark solitons

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Received October 16, 1992

It is shown that a novel class of optical soliton is possible in the form of vector dark solitons. These nonlinear waves describe bound states of two gray solitons with different background intensities in each mode that are strongly coupled through cross-phase modulation.

As is well known, optical pulses may propagate in nonlinear media, e.g., in nonlinear optical fibers,[1,2] without dispersive broadening in the form of bright or dark solitons when nonlinearity, which results from the nonlinear refractive index, exactly compensates for the group-velocity dispersion (GVD). For various optical applications, such as ultralong-distance soliton transmission and switching devices, it is important to understand the nature and general features of the interaction between soliton pulses. The interaction between closely spaced solitons belonging to one optical mode may be understood by analyzing interaction forces between solitons (see, e.g., Refs. 3–6). However, soliton interactions in the case of a few coupled optical modes seem more complicated, and they are not well understood yet. The problem of the intermode interaction of solitons naturally appears, e.g., in highly birefringent optical fibers, as that of nonlinear attraction between pulses of two polarization modes. This effect has been recently studied experimentally and theoretically for possible applications to soliton switching.[7–9]

There exist four principally different cases of the intermode interaction between solitons: (i) two bright solitons propagate in the anomalous dispersion region (see, e.g., Ref. 7), (ii) one bright soliton in the anomalous GVD region interacts with a dark soliton in the normal GVD region (the normal case; see, e.g., Ref. 10), (iii) one bright soliton in the normal GVD region interacts with a dark soliton in the anomalous GVD region (the so-called inverted case; see, e.g., Ref. 11), and (iv) two dark solitons propagate in the normal dispersion region. All these cases may be described by a system of two nonlinear Schrödinger (NLS) equations, coupled as a result of cross-phase modulation. In the case of attraction, partial soliton pulses may form a two-component (vector) soliton as a bound state of the pulses belonging to different polarization modes.[10,12]

Most theoretical and experimental studies have considered the interaction of two bright solitons[7–10] or bright and dark solitons,[10,13,14] but interaction of two dark solitons belonging to different optical modes has not been analyzed yet. However, recent experimental results in which stable propagation of dark solitons has been already observed[15–17] make this problem real.

The purpose of this Letter is to analyze the interaction of two dark solitons coupled through cross-phase modulation and to present a novel class of soliton solutions in the form of vector dark solitons. We also show that the partial dark solitons, which are composed to form the vector dark soliton, are strongly coupled, and in the small-amplitude limit they are described by a single Korteweg–de Vries (KdV) equation.

The interaction of two optical modes, Ψ_1 and Ψ_2, through cross-phase modulation is governed by the system of the incoherently coupled NLS equations[18,19]:

$$i\frac{\partial \Psi_1}{\partial z} + i\frac{1}{v_1}\frac{\partial \Psi_1}{\partial t} - \frac{\alpha}{2}\frac{\partial^2 \Psi_1}{\partial t^2} + R(|\Psi_1|^2 + \sigma|\Psi_2|^2)\Psi_1 = 0, \quad (1)$$

$$i\frac{\partial \Psi_2}{\partial z} + i\frac{1}{v_2}\frac{\partial \Psi_2}{\partial t} - \frac{\alpha}{2}\frac{\partial^2 \Psi_2}{\partial t^2} + R(|\Psi_2|^2 + \sigma|\Psi_1|^2)\Psi_2 = 0, \quad (2)$$

where v_1 and v_2 are the group velocities of the two optical polarization modes and α ($\alpha > 0$) is the coefficient of the normal GVD, which for simplicity we assume equal for both modes. The cross-phase modulation coefficient σ depends on the ellipticity of the fiber eigenmode (see, e.g., Ref. 19) and, in particular, $\sigma = 2/3$ for linearly polarized modes, $\sigma = 2$ for circular polarized modes, and, in the general case, $2/3 \leq \sigma \leq 2$ for elliptical eigenmodes. With the change of variables, $(t - z/v)/t_0 \rightarrow \tau$, $z(\alpha/2t_0^2) \rightarrow \xi$,

$$\Psi_{1,2}\sqrt{2Rt_0^2/\alpha}\,\exp\left(\pm\frac{\delta t}{\alpha} - \frac{\delta^2 z}{2\alpha}\right) \rightarrow U, V,$$

where

$$v = \frac{2v_1v_2}{v_1 + v_2}, \qquad \delta = \frac{v_2 - v_1}{2v_1v_2}, \quad (3)$$

one obtains the system of two dimensionless equations,

$$i\frac{\partial U}{\partial \xi} - \frac{\partial^2 U}{\partial \tau^2} + (|U|^2 + \sigma|V|^2)U = 0, \quad (4)$$

$$i\frac{\partial V}{\partial \xi} - \frac{\partial^2 V}{\partial \tau^2} + (|V|^2 + \sigma|U|^2)V = 0, \quad (5)$$

which describe coupled optical polarization modes operating in the normal GVD region. For $\sigma \neq 1$,

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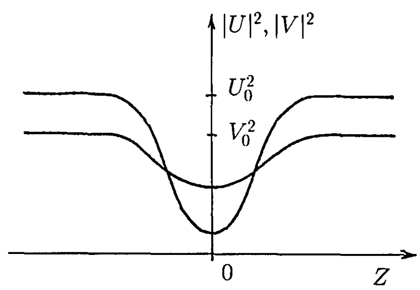

Fig. 1. Intensities of the two components, $|U|^2$ and $|V|^2$, of the vector dark soliton, with U_0 and V_0 being the asymptotic values of the intensities of the orthogonal polarization modes.

these equations are not integrable,[20] but for $\sigma = 1$ they have a set of integrals of motions so that one may naturally expect to find the integrability property and exact soliton solutions as in the case of anomalous GVD.[21]

For the case of anomalous GVD, exact soliton solutions to the system [Eqs. (4) and (5)] may be obtained by a simple substitution, and they are either of equal amplitudes, $U = V$, or of partial orthogonal polarizations, $U = 0$, $V \neq 0$ and $U \neq 0$, $V = 0$. However, in the case of the normal GVD, a general dark soliton solution may have different intensities for two polarization modes. It means that such a solution cannot be obtained by simple substitutions like those mentioned above. However, we have found that the vector dark soliton does exist in the form of two partial dark solitons excited on different cw backgrounds. This solution may be written in the form

$$U = U_0(\cos \phi_1 \tanh Z + i \sin \phi_1)\exp(i\Theta_1), \quad (6)$$

$$V = V_0(\cos \phi_2 \tanh Z + i \sin \phi_2)\exp(i\Theta_2), \quad (7)$$

where $Z = \nu(\tau + \xi/W - \tau_0)$ and

$$\Theta_1 = k_1\tau + (U_0^2 + \sigma V_0^2 + k_1^2)\xi,$$

$$\Theta_2 = k_2\tau + (V_0^2 + \sigma U_0^2 + k_2^2)\xi,$$

and the parameters U_0, V_0, ν, W, k_1, and k_2 are coupled by the following relations:

$$U_0 \cos \phi_1 = V_0 \cos \phi_2, \qquad \nu^2 = \frac{1 + \sigma}{2} U_0^2 \cos^2 \phi_1, \quad (8)$$

$$W^{-1} = U_0\sqrt{\frac{1 + \sigma}{2}}\,\frac{\sin(\phi_1 + \phi_2)}{\cos \phi_2} + (k_1 + k_2),$$

$$k_2 - k_1 = U_0\sqrt{\frac{1 + \sigma}{2}}\,\frac{\sin(\phi_1 - \phi_2)}{\cos \phi_2}. \quad (9)$$

The pulse intensities in each mode may be calculated to be

$$|U|^2 = U_0^2\left(1 - \frac{\cos^2 \phi_1}{\cosh^2 Z}\right),$$

$$|V|^2 = V_0^2\left(1 - \frac{\cos^2 \phi_2}{\cosh^2 Z}\right). \quad (10)$$

Solutions (6) and (7) describe two coupled dark (gray or black; see the terminology in Ref. 22) solitons in the case for which the asymptotic values of the cw backgrounds are different, i.e., $|U| \to U_0$ and $|V| \to V_0$ provided that $|\tau| \to \infty$ (see Fig. 1).

The partial dark solitons described by the solutions (6) and (7) are indeed strongly coupled owing to mutual trapping. To demonstrate such a trapping analytically, we use a variational (Lagrangian) approach[14] to introduce a time delay into one of the pulse components, e.g., by putting

$$|V(Z)|^2 \to V_0^2\left[1 - \frac{\cos^2 \phi_2}{\cosh^2(Z - \Delta)}\right]. \quad (11)$$

Then the pulse interaction results in the system Lagrangian from the integral,

$$\int_{-\infty}^{\infty} d\tau(|U|^2 - V_0^2)(|U|^2 - U_0^2),$$

which yields the effective interaction energy between the soliton pulses,

$$U_{\text{eff}}(\Delta) = -4\sqrt{2}U_0^3 \frac{\sigma \cos^3 \phi_1}{\sqrt{1 + \sigma}} \frac{\cosh \Delta}{\sinh^3 \Delta}(\Delta - \tanh \Delta). \quad (12)$$

The effective interaction energy [Eq. (12)] has a sense either for small σ (small intermode coupling) when $\Delta \sim 1$ or for small Δ (small time delay) when $\sigma \sim 1$, and it corresponds to an effective attraction of partial dark solitons.

The strong coupling between partial dark solitons may be also confirmed in another way, by considering the small-amplitude limit of the vector soliton [Eqs. (6) and (7)]. Following to the approach developed for the one-component NLS equation,[23] we look for solutions of the coupled system (4) and (5) in the form

$$U = (U_0 + a)\exp[i(U_0^2 + \sigma V_0^2)\xi + i\phi], \quad (13)$$

$$V = (V_0 + b)\exp[i(V_0^2 + \sigma U_0^2)\xi + i\psi], \quad (14)$$

assuming that a, b and derivatives of ϕ and ψ are small. Substituting Eqs. (13) and (14) into the system (4) and (5), one may obtain four coupled equations for the functions a, b, ϕ, and ψ. This system of equations may be investigated by the asymptotic method,[23] and it shows that even in the small-amplitude limit the optical modes are strongly coupled if waves propagate along the finite-amplitude backgrounds. In the linear limit such excitations are described by the dispersion relation

$$(K^2 - 2U_0^2\Omega^2 - \Omega^4)(K^2 - 2V_0^2\Omega^2 - \Omega^4) = 4\sigma^2U_0^2V_0^2\Omega^4, \quad (15)$$

with K and Ω being the wave number and wave frequency, respectively, of linear waves. In this approximation, the modes dynamics may be simplified by introducing normal variables as linear combinations of a and b. This result simply means that for $\sigma \sim 1$ any dip on the cw background pulse of one mode will immediately create a similar dip in the other mode strongly coupled to the primary pulse. This situation drastically differs from the case of intermode interaction of bright solitons.

Such a strong interaction between the polarization modes also remains valid in the nonlinear case, and in the limit of small amplitudes the mode dynamics is described by a single KdV equation but not by the coupled equations. For example, in the case $U_0 = V_0$, the asymptotic expansions (which are similar to those for the one-component NLS equation[23]) yield the simple relation $a = b$, and in the zero-order approximation the evolution of the pulse amplitude a is given by the KdV equation

$$2C \frac{\partial a}{\partial y} + 12U_0(1 + \sigma)a \frac{\partial a}{\partial T} - \frac{\partial^3 a}{\partial T^3} = 0, \qquad (16)$$

where slow variables y and T (see details in Ref. 23) are connected with the reference frame moving at the sound velocity C, with $C^2 = 2(1 + \sigma)U_0{}^2$.

The strong coupling between the partial dark pulses means that optical switching involving dark pulses is not possible in the way already proposed for bright solitons.[24] To support such a conclusion by a clear example, let us consider the model of a nonlinear directional coupler (see, e.g., Ref. 25 and references therein), which in dimensionless units is described by the coupled equations

$$i \frac{\partial U}{\partial \xi} - \frac{\partial^2 U}{\partial \tau^2} + (|U|^2 + \sigma |V|^2)U = -\kappa V, \qquad (17)$$

$$i \frac{\partial V}{\partial \xi} - \frac{\partial^2 V}{\partial \tau^2} + (|V|^2 + \sigma |U|^2)V = -\kappa U. \qquad (18)$$

In the case $\sigma = 1$ the system (17) and (18) has exact solutions (which may be found analogously to the bright soliton case[26]); the simplest one is

$$|U|^2, |V|^2 = \frac{1}{2} U_0{}^2[1 \pm \sin(2\beta)\cos(2\kappa\xi)]\tanh^2(U_0\tau), \qquad (19)$$

where β is an arbitrary constant. The result [Eq. (19)] looks like that of the linear theory except for the τ-dependent factor describing the dark-profile envelope. Thus, as follows from Eq. (19), the optical switching in this case is realized through background pulses themselves but not through dark solitons.

In conclusion, we have found a new class of solutions of the two coupled NLS equations in the form of a vector dark soliton that describes mutual trapping of two dark-profile pulses. We have shown that these waves correspond to strongly coupled states that in the small-amplitude limit may be described by a single KdV equation. We have also pointed out that a strong coupling of the partial solitons with their backgrounds does not allow one to use dark solitons for optical switching in the way already proposed for bright solitons.

The authors are indebted to K. H. Spatschek for warm hospitality at the Institut für Theoretische Physik I. This research has been supported by the Alexander von Humboldt-Stiftung.

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