Supplementary Information
Unveiling temporal correlations characteristic to phase transition in the intensity of a fibre laser radiation

A. Aragoneses,[1,2] L. Carpi,[1] N. Tarasov,[3,4] D. V. Churkin,[3,5,6] M. C. Torrent,[1] C. Masoller,[1] and S. K. Turitsyn[3]

[1] *Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, 08222 Terrassa, Spain*
[2] *Duke University, Physics Department, Box 90305, Durham, North Carolina 27708, USA*
[3] *Aston Institute of Photonic Technologies, Aston University, UK*
[4] *Institute for Computational Technologies, SB RAS, Novosibirsk 630090, Russia*
[5] *Institute of Automation and Electrometry SB RAS, Novosibirsk, Russia*
[6] *Novosibirsk State University, Russia*

In order to demonstrate the robustness of the results with respect to the choice of the threshold, in Fig. 1 we plot the OP probabilities with $D = 2$ and $D = 3$ and the PE-entropy for three thresholds: 0, 1, and 2 (i.e., as the time-series are normalized to zero mean and unit standard deviation, with these thresholds we consider only the intensity peaks that are above the mean value, above one standard deviation, and above two standard deviations). In the three cases one can observe that the same transition is detected, at the same value of the control parameter.

Figure 2 displays the HVG-entropy, for the same thresholds. Here we can observe the same transition, confirming the robustness of the results with respect to the choice of threshold. HVG entropy is normalized to that of the Gaussian white noise, which rapidly converges to stable values for increasing time series lengths (percentage variations for times series with $N = 10^5$ and $N = 5 \times 10^5$ data points are 10^{-4} and 10^{-5}; in contrasts, the normalization through $\log(N)$ results in decreasing entropy values as N increases [1].

In the case of ordinal analysis, as Fig. 3 shows, the results are robust also with respect to the length of the ordinal pattern, D. Larger D values were not considered due to the finite length of the dataset: while the "raw" intensity time series contains 50 million data points, the number of high intensity peaks (above 2σ), as shown in Fig. 1(d), is about 10^5, depending on the pump power.

Error bars in Fig. 2 (b) of the main text are computed with a binomial test, and indicate a confidence level of 95%; the "gray region" indicates probability values that are consistent with a uniform distribution (i.e., consistent with no serial correlations in the data) [2]: it indicates probabilities in the interval $p \pm 3\sigma$ where $p = 1/D!$ and $\sigma = \sqrt{(p(1-p)/N)}$ with $D!$ being the number of possible patterns and N being the number of ordinal patterns.

[1] M. Gomez Ravetti, L. C. Carpi, B. Goncalves, A. C. Frery and O. A. Rosso, PLOS ONE **9**, e108004 (2014).
[2] A. Aragoneses, S. Perrone, T. Sorrentino, M. C. Torrent, and C. Masoller, Sci. Rep. **4**, 4696 (2014).

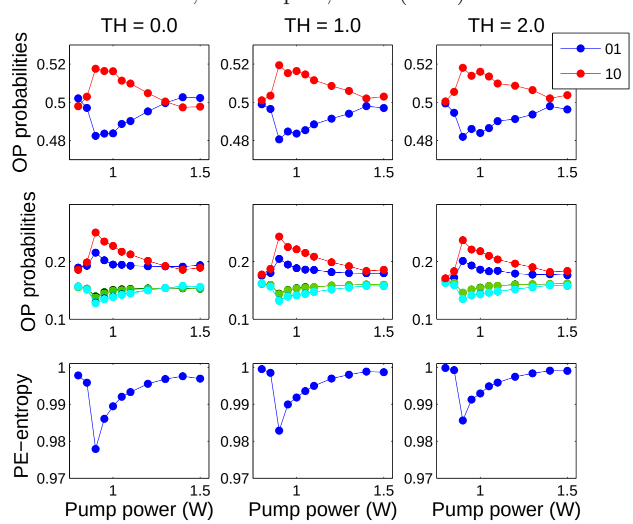

FIG. 1. Ordinal pattern probabilities with $D = 2$ (top row), $D = 3$ (center row) and the PE-entropy (bottom row) vs. the experimental control parameter (the laser pump power). Three thresholds are used in order to filter out the small intensity pulses: 0 (left column), 1 (center column), and 2 (right column).

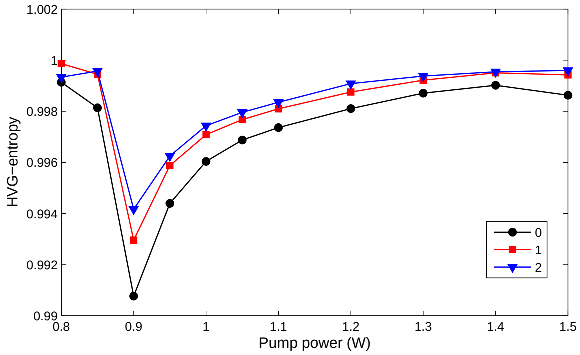

FIG. 2. HVG-entropy vs. the laser pump power. The thresholds used are as in Fig. 1.

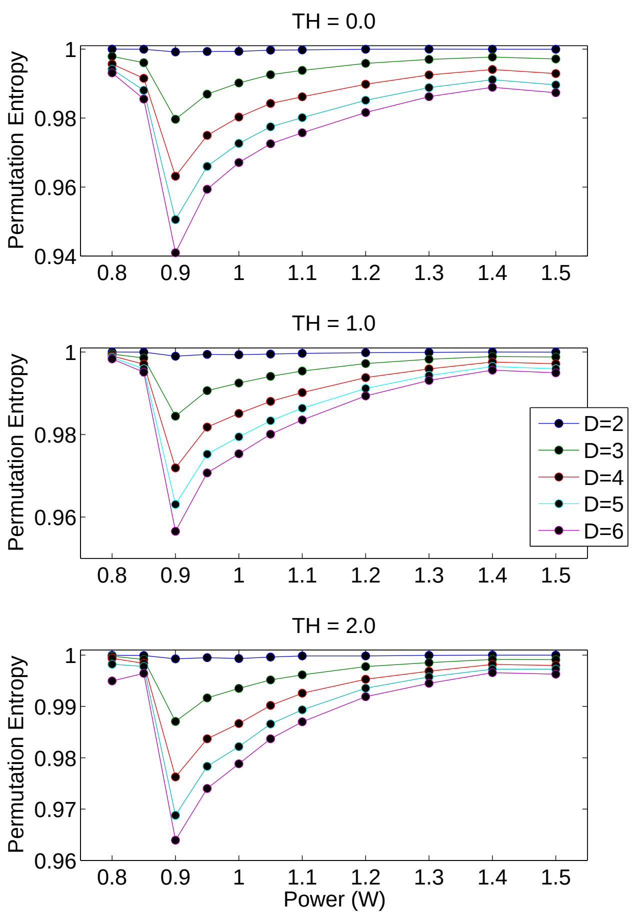

FIG. 3. Permutation entropy vs. the laser pump power for various D values; the thresholds used are as in Figs. 1 and 2.