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Soliton content in the standard optical OFDM signal

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The nonlinear Schrödinger equation (NLSE) is often used as a master path-average model for fiber-optic transmission lines. In general, the NLSE describes the co-existence of dispersive waves and soliton pulses. The propagation of a signal in such a nonlinear channel is conceptually different from linear systems. We demonstrate here that the conventional orthogonal frequency-division multiplexing (OFDM) input optical signal at powers typical for modern communication systems might have soliton components statistically created by the random process corresponding to the information content. Applying the Zakharov–Shabat spectral problem to a single OFDM symbol with multiple subcarriers, we quantify the effect of the statistical soliton occurrence in such an information-bearing optical signal. Moreover, we observe that at signal powers optimal for transmission, an OFDM symbol incorporates multiple solitons with high probability. The considered optical communication example is relevant to a more general physical problem of the generation of coherent structures from noise.

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An optical fiber is a remarkable engineered physical medium important for a range of practical applications, including telecommunications, sensing, lasers, and imaging [1]. Light trapped in silica waveguide can propagate with extremely low-field attenuation over long distances. An optical fiber medium can also act as a nonlinear system when a signal accumulates a noticeable (of the order of π) nonlinear phase change due to the fiber Kerr effect during the propagation. In some applications, such as mode-locked fiber lasers, the nonlinear Kerr effect is used positively, providing conditions for mode locking and pulse shaping in a laser. In modern telecommunication systems, nonlinearity is typically considered as the factor limiting their performance at a high signal-to-noise ratio.

The nonlinear properties of the fiber communication links create a number of unusual (compared to linear channels) challenges. However, channel nonlinearity also offers new interesting opportunities. It is well known that the nonlinear Schrödinger equation (NLSE) describes under particular conditions and within certain limits the propagation of a signal down an optical fiber [1–5]. Written in the generic normalized form (for details, see Refs. [1–3]) the NLS equation reads

$$i\frac{\partial q}{\partial z} + \frac{1}{2}\frac{\partial^2 q}{\partial t^2} + |q|^2 q = 0. \tag{1}$$

In the context of fiber-optic, we consider here the case of the so-called anomalous dispersion, when general solutions of this equation can include both the dispersive (linear-like) waves and the coherent structures—solitons. Any initial field evolving according to this master model can be presented as a nonlinear superposition of dispersive waves and soliton(s).

In 1972, Zakharov and Shabat demonstrated that the NLSE can be integrated by the inverse scattering transform (IST) method [6], also known today as the nonlinear Fourier transform (NFT). The IST/NFT method allows one to present a field (with the evolution along distance z governed by NLSE) at an arbitrary distance using nonlinear spectrum of the initial (at $z = 0$) signal distribution. More specifically, the nonlinear spectrum of the initial field $q(t, z = 0)$ can be found through the solution of the Zakharov–Shabat spectral problem:

$$\begin{cases} -\partial_t \psi_1 + q(t,0)\psi_2 = i\xi\psi_1 \\ \partial_t \psi_2 + q^*(t,0)\psi_1 = i\xi\psi_2 \end{cases}. \tag{2}$$

$q(t, 0) = q_0(t)$ is the "potential"—initial distribution of the signal; $\psi_{1,2}$ is a vector eigenfunction; and ξ is the spectral parameter defined on a complex plane.

In general, the nonlinear spectrum for the localized in time domain optical signal $q_0(t)$ has discrete eigenvalues and a continuous component corresponding to the spectrum of the system (2). The continuous spectrum of the system (2) fills the real axis of the ξ-plane and corresponds to the dispersive wave component, which will not be our focus here.

The discrete spectrum eigenvalues ξ_n correspond to soliton solutions of the NLSE. For some classes of initial pulses, there are known analytical and numerical results concerning the soliton content in the initial field $q_0(t)$; for details, see Refs. [7–9]. In the case of a real-valued unmodulated (no temporal dependence of the phase) rectangular pulse, the number of solitons N contained in such a field can be found as

$$N = \mathrm{int}[1/2 + L_1(q)/\pi], \tag{3}$$

where int[…] means the integer part of the expression, and $L_1(q) = \int_{-\infty}^{+\infty} |q(t)| \mathrm{d}t$ is the (non-dimensional) signal L_1 norm. Evidently, the L_2 norm given by $L_2 = \int_{-\infty}^{+\infty} |q(t)|^2 \mathrm{d}t$ corresponds to the signal energy. In the case of more complex initial signals, the analytical approaches are limited, and the analysis requires extensive statistical numerical modeling based on direct solving of the Zakharov–Shabat spectral problem (2). For instance, in Ref. [10], the generation of solitons from noise and noncoherent optical pulses has been considered using L_2 norm as a measure. The analysis of a soliton content in chirped Gaussian pulses was done in Ref. [11] and in the optical speckle fields in Ref. [12]. In particular, it was shown that modulation of a simple rectangular pulse leads to significant decrease of the number of emerging solitons [10].

It is well known that an information-bearing signal can be treated as a random process in which signal characteristics, such as power and phase, experience statistical variations that depend on modulation formats and coding [13]. Here we study the soliton content in a standard optical orthogonal frequency-division multiplexing (OFDM) signal, in which digital data are encoded on multiple carrier frequencies. Here we are interested only in the total number of discrete eigenvalues, not in their specific parameters. Therefore, we apply the method described in Ref. [14], which links the number of discrete eigenvalues to the total phase shift of the coefficient $a(\xi)$ by the formula

$$N = \frac{1}{2\pi} \mathrm{Arg}(a(\xi))|_{-\infty}^{+\infty}, \tag{4}$$

where the spectral parameter ξ takes values from $-\infty$ to $+\infty$ on the real axis. The coefficient $a(\xi)$ is one of the coefficients characterizing the scattering on the "potential" $q(t, 0)$ in the Zakharov–Shabat problem (2); for details, see Ref. [15] and the recent work, Ref. [16].

We would like to stress that in the framework of the considered integrable NLS equation model there is no need for numerical simulations of the initial signal propagation with distance z. The number of discrete eigenvalues will not be changed during the propagation governed by the NLS equation. Moreover, the parameters of continuous and discrete nonlinear spectra will be changed in a trivial manner [5]. Therefore, we focus in this Letter on the analysis of the solutions of the Zakharov–Shabat spectral problem, rather than on the consideration of the propagation dynamics of the field.

OFDM combines multiplexing and modulation. A single OFDM symbol (over a time interval with duration T) is presented as a sum of independent subcarriers:

$$s(t) = \sum_{k=0}^{M-1} X_k e^{i2\pi kt/T}, \qquad 0 \le t < T \tag{5}$$

where X_k corresponds to the digital data, M is the number of subcarriers, and T is the symbol interval. In practice, the number of subcarriers is selected as $M = 2^p$ to use the fast Fourier transform (FFT) algorithm. In the real world units, we examine the OFDM symbol with 10 ns symbol duration and quadrature phase-shift keying (QPSK) or 16 quadrature amplitude modulation (QAM). The full FFT size is 1024, and the number of subcarriers M is changing from 16 to 1024. The average signal power is linked to the L_2 norm (in dimension units) as follows: $P_{\mathrm{ave}} = L_2/T$, and it varies (in the dimension units) from −21 to −8 dBm.

Without the loss of generality, we focus here on two types of popular modulation formats: QAM and phase-shift keying (PSK), and consider a single OFDM symbol (i.e., assuming burst mode transmission with well separated symbols). We analyze the probability of the appearance of solitons in the input OFDM symbol, depending on the signal parameters: modulation type, L_1 or L_2 norms, and the number of sub carriers M. We use in the numerical simulations shown in Fig. 5 the following typical parameters: group velocity dispersion parameter $\beta_2 = -21.5$ (in ps^2/km) and the nonlinear Kerr coefficient $\gamma = 1.27$ (in W^{-1} km^{-1}). We accumulate statistics on the number of occurred solitons for a fixed system and signal parameters by varying the input digital data. Each graph point corresponds to 160 statistical measurements. For example, Fig. 1 shows the probability distributions for an OFDM signal with QPSK modulation at 128 subcarriers with $P_{\mathrm{ave}} = -18$ dBm and Poisson fit distribution ($P(x; \lambda) = e^{-\lambda} \cdot \lambda^N/N!$) for this data ($\lambda$ is extracted from the data fitting), obtained by numerical simulation. The number of events equals 10^6.

We now examine the probability of the occurrence of solitons in the OFDM signal defined as the ratio of the number of symbols containing discrete eigenvalues (corresponding to solitons) of the Zakharov–Shabat spectral problem to the total number of the considered OFDM symbols. In other words, we are not interested in the exact number of solitons in the signal, but rather in their existence in the given OFDM symbol. Our goal here is to demonstrate that the appearance of solitons in the OFDM signal is not something exotic, but rather is a general situation at certain practical power levels. We verified that the number of solitons does not depend on an increase of

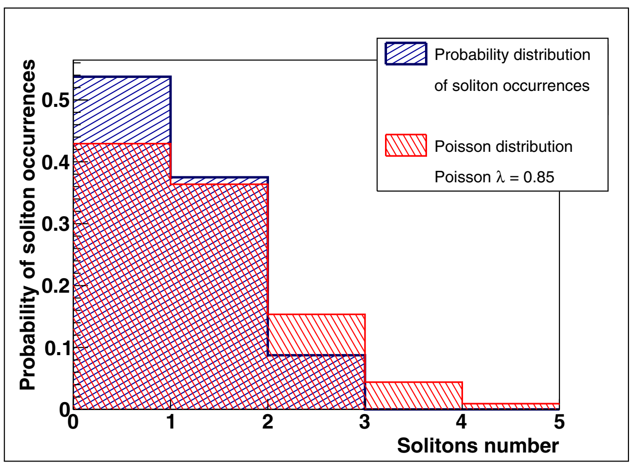

Fig. 1. Probability distribution of soliton occurrences in the OFDM symbol with QPSK modulation, with 128 subcarriers and an average power of −18 dBm.

the computational grid and the FFT size (temporal signal discretization).

Figure 2 shows the probability of occurrence of solitons at 128 subcarriers versus the L_1 norm value. Note that the probability of the soliton appearance for signals with the same L_1 norm value is higher for signals with 16QAM modulation compared to QPSK. This trend is maintained for all numerical modeling with various parameters. It is also seen that for such highly modulated complex signals, the threshold of the soliton occurrence is much higher than the analytical results (3) obtained for real unmodulated rectangular smooth functions.

Figures 3 and 4 show how the probability of the occurrence of the soliton content in the OFDM signal is growing with the increase of the average power P_{ave}. One can see that in signals

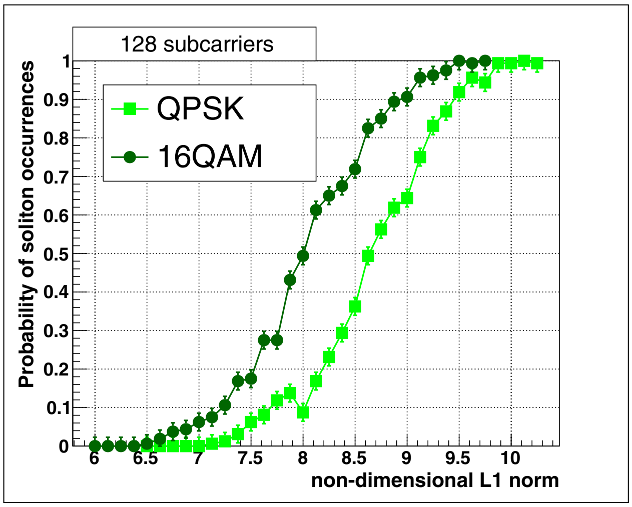

Fig. 2. Average number of occurred solitons versus the value of L_1 norm for OFDM signals with QPSK and 16QAM modulations. The first threshold of the L_1 norm value, calculated using the formula (3), is 1.57 and lies on the left, well outside the boundaries of the graph.

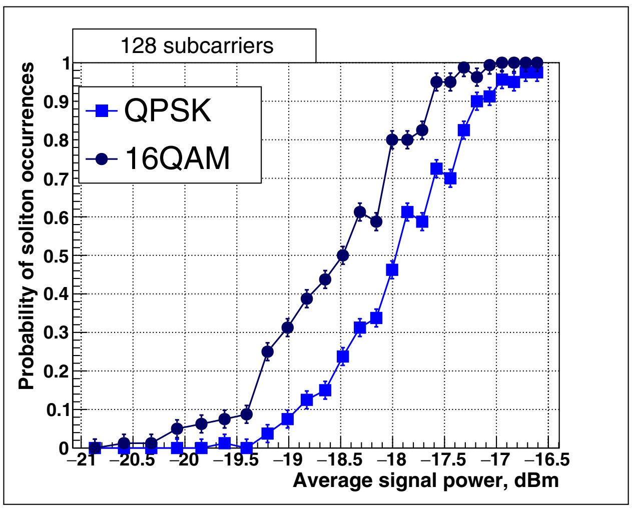

Fig. 3. Average number of solitons embedded into the OFDM symbol with 128 subcarriers and QPSK and 16QAM modulations versus the average signal power.

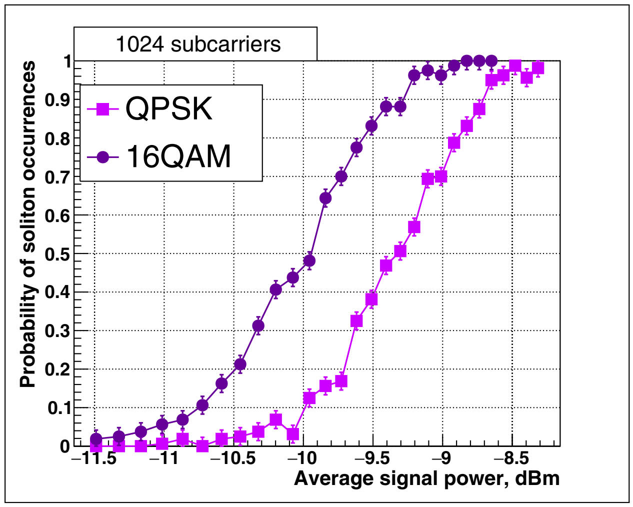

Fig. 4. Average number of solitons embedded into the OFDM symbol with 1024 subcarriers, and QPSK and 16QAM modulations versus the average signal power.

with 128 subcarriers, solitons start to emerge at lower values of the norm compared to signals with 1024 subcarriers. It is seen that this effect depends on the number of subcarriers and the signal modulation format. The transition from random (depending on the information content) appearance of solitons in certain (relatively rare) OFDM signals to the regime where most of the symbols contain discrete eigenvalues (soliton component) happens over the interval of a 3–4 dB increase of input signal power. An interesting observation is that the soliton component does not require signal power that is too high to become an inherent part of the OFDM symbol. The soliton component arises at rather practical levels of a signal power conventional for telecom applications.

We would like to stress that solitons appear in the OFDM signal at powers that are not very high. As a matter of fact, a soliton content is present in the OFDM signals at the powers optimal for transmission. To illustrate this point, we considered 1000 and 2000 km transmission links based on an ideal distributed Raman amplification scheme with continuous amplified spontaneous emission generation (see Refs. [4,5] for details). As an input, we used 16QAM-OFDM signals with 128 subcarriers and 10 ns symbol duration. At the receiver, the chromatic dispersion was fully compensated for, and an algorithm based on the 4th-power Viterbi–Viterbi method was used for phase estimation. The system performance was evaluated using the parameter Q^2-factor, which measures the quality of a transmission signal. The Q^2-factor value has been extrapolated from the conventional error vector magnitude function [17] as $Q^2 = 1/\mathrm{EVM}^2$ using the transmission of 2^{14} OFDM symbols per run. The results are shown in Fig. 5. Optimal transmission is achieved with an average power around −15 dBm which, according to Fig. 3, is well in the regime where a soliton component in the OFDM symbol is likely high.

In general, solitons and dispersive waves propagate in a different manner down the optical fiber. The most noticeable difference is that in the soliton, dispersive broadening is counterbalanced by the nonlinear effects. Therefore, the presence of solitons embedded into the conventional OFDM

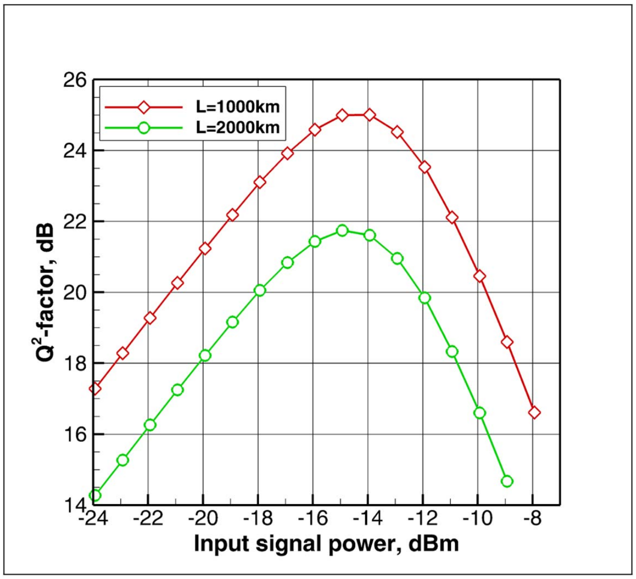

Fig. 5. Dependence of the Q^2-factor on the average input power of the signal.

symbol potentially should impact the transmission of the combined signal. Comprehension of this fact and its consequences for signal coding, modulation, and processing might be important for the improvement of the performance of fiber-optic communication systems. However, it should be pointed out that the effect of the embedded solitons does not lead to a drastic change of the symbol propagation dynamics. Further study is required to understand how the presence of solitons affects transmission and how it can be used in practical terms. Note that even for quasi-linear signal propagation, statistical fluctuations in the plane (z, t) might be very nontrivial and are dependent on the information data, format, and modulation; see, e.g., the recent publication Ref. [18].

Traditional signal modulation formats have been designed and developed for linear communication channels. The transmission in the nonlinear channel reveals rather unusual properties of such conventional signals. Considering the NLSE as a master channel model, we have shown here that a standard OFDM signal statistically contains soliton components at the powers of practical interest. Using the Zakharov–Shabat spectral problem, we studied the statistics of soliton occurrences in a OFDM signal and quantify how the number of solitons that are embedded into the input OFDM signal increases with the L_1 norm and signal average power.

This observation indicates that transmission in a nonlinear channel substantially changes the whole paradigm of signal modulation and processing. Our results show that nonlinear analysis might be useful not only for special inherently soliton-based systems and signals [3], but also for conventional communication formats that traditionally are not linked to the soliton concept and techniques. We believe that our results indicate that the application of the detection and processing methods developed for linear channels might not be optimal for nonlinear communication channels. In this Letter, our focus was on proving the fact of the occurrence of solitons in the OFDM signal and study of the statistics of soliton component appearances. The impact of such low energy solitons on signal dynamics and, overall, on the transmission will be examined elsewhere.

Note that our Letter is also relevant to the recently restarted studies of the so-called integrable turbulence (see, e.g., Refs. [19–21] and references therein), where a random initial signal q_0 (e.g., amplified spontaneous emission) evolves in an intricate way in the plane (z, t) according to (1). A statistical analysis of this evolution presented in the nonlinear spectrum can provide new insights in the complex dynamics of the optical field.

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