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EDITORIAL

Recent progress in investigating optical rogue waves

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The science of rogue waves in optics is now over five years old, and it has emerged as an area of broad interest to researchers across the physical sciences [1]. This area of study was initiated by the pioneering measurement of Solli *et al* [2] when analysing supercontinuum generation in optical fibres. Their measurements, using a novel dispersive Fourier transform technique to capture high-speed events in the time domain, observed extraordinarily high amplitude peaks at certain wavelengths in the chaotic spectrum from the supercontinuum. By analogy with the extreme waves in the ocean [3], of wide interest after 1995, such high amplitude pulses were described as ‘optical rogue waves’.

This analogy between localized structures in optics and extreme waves on the ocean has opened up many possibilities for exploring extreme value dynamics in convenient table-top optical experiments. In addition to the proposed links with solitons suggested in [2], other recent studies, motivated from an optical context, have explored possible links with nonlinear breather propagation. There is now an international effort, worldwide, to study these extreme events in optics, both for their own intrinsic interest within their own domain of research, and also because of their links with the large amplitude ocean wave events [4] that have inspired their study.

This subject has been rapidly expanding since 2007, with many published works in different fields: supercontinuum generation in a fibre [5–21], nonlinear waves in optical cavities [22, 23], pulsed operation of passively mode-locked lasers [24–28] and wave propagation in photonic crystal fibres [29]. Rogue waves have been found in erbium-doped fibre systems [30], Raman fibre amplifiers [31, 32], as spatiotemporal structures [33], in parametric processes [34], when analysing phase profiles [35] and in studies of ways of stimulating and moderating the appearance of extreme fluctuations [6, 13, 15, 36–41]. Insights obtained from these works in optics have also motivated parallel work in hydrodynamics to explore, in more detail, large amplitude waves in their ‘original’ environment [42].

A number of theories for optical rogue wave formation have been advanced for different experimental conditions. The appearance of rare or unexpected events, as considered here, is a classical and thus deterministic process. Anomalous events nevertheless arise due to sensitivity to the initial conditions. Modulation instability, a complex nonlinear process exhibiting emergent behaviour and strong sensitivity to initial conditions [16, 18], has been found to play a central role in the appearance of rogue waves in many optical [2, 8, 18, 43] and hydrodynamic scenarios. In the case of supercontinuum generation [44, 45], rogue waves can appear as rare solitons, possessing anomalously large red-shifted energy and peak intensity. Such rare solitons arise when modulation instability is spontaneously seeded by a surplus in a particular input noise component [2]. The discovery of this process has initiated work demonstrating that long-pulse supercontinuum generation can be stabilized and enhanced by controlled seeding [6, 13]. The influences of added input modulation [5, 20, 46] and feedback [14, 17, 48] on supercontinuum generation have also been considered. In other noise-seeded

situations, gain competition between input noise components can frustrate the formation of a soliton, leading to rare low amplitude events in a pulse train [15].

Under other experimental conditions, rogue waves may arise through soliton collisions, which can produce events with high red-shifted energy [47, 49]. The energy exchange between the solitons is facilitated by third-order dispersion [50, 51] and Raman effects. From multiple collisions, the strong solitons have a tendency to accumulate energy from the weaker ones, thus exiting the fibre as rogue waves.

In a different theoretical description, the initial phase of modulation instability has been shown to lead to the emergence of structures of Akhmediev breathers [43, 52]. The particular limiting case of the Peregrine solution [53] of the nonlinear Schrödinger equation has been seen in controlled experiments aiming to excite the particular prototype isolated rogue wave possessing the strongest localization [54, 55]. Yet, the initiation of these solutions stochastically, which is needed to establish a link with rare events (i.e., rogue waves), remains an open question. Excitation of an isolated rogue wave in a variety of situations is one of the developing areas of research [56–58]. A recently coined phrase, ‘deterministic rogue wave’, stresses this aspect of study [59, 60].

When excited from special initial conditions, we can expect excitation of higher-order rogue wave structures [61–63] described by the higher-order rational solutions of the nonlinear Schrödinger equation [64–69] or its extensions such as the Hirota equation [70–72], Sasa–Satsuma equation [73], the set of coupled nonlinear Schrödinger equations [60, 74–76] and a variety of other equations [77–85]. When the equation is not integrable, approximate solutions in the form of rogue waves can also be found [86].

An interesting mathematical result from these studies is the prediction of circular ‘atomic-like’ structures [87, 88] that still need detailed physical explanations and deserve experimental verification. An optical setup would be an ideal environment for these observations. Generally, multi-parameter higher-order rogue wave solutions can be revealed in a variety of other regular shapes [89, 90]. In particular, triangular rogue wave cascades [91] can be observed using higher-order modulation instability effects [92]. Rogue waves have also been studied in discrete structures [93]. An array of optical waveguides is one example of their application [94].

A further analogy with those ocean rogue waves that appear from a chaotic wave field comes from the notion of ‘optical turbulence’. Wave turbulence is a classical nonlinear phenomenon that is observed in a variety of physical systems. Wave turbulence theory deals with the statistical behaviour of a large number of weakly interacting waves with random phases [95]. In optics, turbulent-type interactions between a very large number of cavity modes may be responsible for some observable characteristics of Raman fibre lasers [96–98], such as spectral broadening of the radiation that is generated. There are several studies stressing this aspect of optical rogue wave generation [47, 99–101].

In systems with an external pump, such as lasers, the models are again different. Individual pulses are amplified by a pump and simultaneously the number of them in the cavity increases. Depending on the type of mutual interaction between the pulses, they tend to bunch into a group, or spread across the whole cavity [26]. If bunching takes place, the solitons collide and reveal higher amplitudes at the time of collision. Multiple collisions may increase the amplitude of output pulses to the size of a rogue wave that is well above the average [8, 27]. As dynamical systems with gain and loss are known as ‘dissipative systems’, extreme pulses generated in these systems have been dubbed ‘dissipative rogue waves’ [24].

A signature of rogue waves is their probability of occurrence, which is larger than expected from standard Gaussian statistical models [2, 22, 102–107]. The

overall impact and significance of occurrence of such high amplitude waves are determined by the nature of the physical system in which they occur. For example, oceanic rogue waves are infamous for their potential to destroy or damage vessels. On the other hand, optical rogue waves have already been suggested for applications in enhancing supercontinuum generation and have elucidated key aspects of nonlinear dynamical processes, as described above.

In recent studies, complete single-shot spectra have been measured at megahertz rates [16]. The large volume of data has revealed the existence of emergent behaviour in modulation instability [16] and yielded new insights into noise-driven supercontinuum generation [18]. The technique has also been used to generate higher-order statistical moments of the chaotic spectral fluctuations [108]. It is interesting to consider that, when the number of statistical realizations becomes large, even Gaussian statistics can provide some probability of occurrence for events with atypical amplitudes.

A signature feature of rogue waves is their anomalously large amplitudes. They involve intensities when nonlinear properties of materials play a significant role. However, when considering chaotic wave fields and probabilities of the highest amplitudes in two-dimensional patterns, linear mechanisms can also be taken into account [109]. Another example comes from signal streams in optical communication links. As was first shown in [110], and then further elaborated in the following discussion [111, 112], waves with amplitudes much larger than the average level can be observed during a short period of time in purely linear propagation regimes in optical fibre systems. Large broadening of short optical pulses due to fibre dispersion leads to strong bit-overlapping in data streams, resulting in statistical deviations of local power from the average level. This linear effect, leading to the random appearance of high amplitude waves, is routinely observed in wireless and optical communications and can have a direct impact on system performance.

The notion of rogue waves has lately expanded to many fields in science [1]. Careful studies on small scales may help to better understand rogue waves in the ocean. The analogy is mainly based on similar equations used to model rogue waves in various fields, including waves in the open ocean. However, specific features of waves in a laboratory also allow them to be considered as individual new directions in science. It would be hard to cover all these directions in a single volume. Nevertheless, the papers presented in this special issue present a wide range of topics. Below, we briefly introduce each of these works.

The paper by DeVore *et al* [113] revisits recent work on the nonlinear dynamics of the generation of extreme optical events in silicon waveguides. The authors show that the underlying processes, namely modulation instability and stimulated Raman scattering, reshape normally distributed initial conditions into states with skewed output statistics with properties that can be tailored by controlling experimental variables. An important observation made by the authors is the fundamental difference between contributions caused by modulation instability and stimulated Raman scattering.

The manuscript presented by Wabnitz [114] stresses deep analogies between optical and ocean nonlinear waves. The term ‘optical tsunamis’ introduced by the author is a direct consequence of this analogy. He presents exact Riemann wave solutions of optical shallow water equations and shows that they agree remarkably well with the numerical solutions of the nonlinear Schrödinger equation. He also reveals that extreme wave events or optical tsunamis may be generated in dispersion-tapered fibres in the presence of higher-order dispersion.

The paper ‘Electromagnetic rogue waves in beam–plasma interactions’ [115], written by Veldes *et al*, presents the results of an investigation into the occurrence of rogue waves associated with electromagnetic pulse propagation interacting with a plasma. The authors solve the fluid-Maxwell equations which describe

weakly nonlinear, circularly polarized, electromagnetic pulses in magnetized plasmas. Various solutions are presented as potential candidates for modelling rogue waves in beam–plasma interactions.

An experimental paper on optical rogue waves in an all-solid-state laser with a saturable absorber [116], by Bonazzola *et al*, stresses the importance of spatial effects. The authors study the features of the optical rogue waves observed in an all-solid-state Cr:YAG + Nd:YVO₄, passively *Q*-switched, laser. The extreme events appear as isolated pulses of extraordinary intensity during the chaotic regime of this laser. Interestingly, existing standard laser models fail to predict rogue waves. The authors have found that extreme pulses are observed only when there are high values of the Fresnel number of the laser cavity and the embedding dimension of the attractor reconstructed from the experimental time series.

The work of Lecaplain *et al* [117] provides an extensive study of the experimental conditions under which dissipative rogue waves, generated in a laser cavity, can be detected. The authors have found that rogue waves originate from the nonlinear interactions of bunched chaotic pulses that propagate in a fibre laser cavity. They appear as rare events of high optical intensity. The crucial influence of the electrical detection bandwidth is one of the important observations in this paper.

Bandelow *et al* study the solutions of Sasa–Satsuma equation that start from a continuous wave field [118]. This equation contains three terms that are important for wave propagation in optical fibres. These are the third-order linear dispersion, a higher-order nonlinear dispersion and the Raman effect. In the case of arbitrary coefficients for each term, the only way to find solutions of the equation is numerical modelling. For the Sasa–Satsuma equation, the coefficients have to be related in a special way. Despite this serious restriction, having analytic solutions, even in special cases, can be quite helpful in building knowledge related to some of the features of short-pulse propagation in optical fibres. Such an analytic rogue wave solution is an important case.

When the coefficients in the NLSE with the above-mentioned terms are arbitrary, analytic studies can be based on various approximations. One example of such a study is the paper by Ankiewicz *et al* [119]. The authors show that a rogue wave solution of the nonlinear Schrödinger equation (NLSE) can survive even-parity perturbations of the equation, such as the addition of a quintic term and fourth-order dispersion. They present a solution which is accurate to the first order for such a perturbation.

The paper on rogue waves and related solutions of single and coupled Ablowitz–Ladik and nonlinear Schrödinger equations by Ankiewicz *et al* [120] provides a simple technique for finding the correspondence between the solutions of Ablowitz–Ladik and nonlinear Schrödinger equations. Even though they belong to different classes, in that one is continuous and one is discrete, there are matching solutions. Several examples are presented, including the rogue wave solutions. This technique is also extended to the case of coupled Ablowitz–Ladik and Manakov equations.

The authors of [121] present a numerical study devoted to generation of extreme events in lumped Raman fibre amplifiers. They analyse the evolution of signals, taking into account cross-correlations, the spectra obtained and probability density functions for high-energy pulses, in addition to exploring their phase evolution.

Bludov *et al* [122] study the effect of modulational instability of a continuous wave and the subsequent generation and evolution of deterministic rogue waves in a parity-time (PT)-symmetric system of linearly coupled nonlinear Schrödinger equations. This system describes a Kerr-nonlinear optical coupler with mutually balanced gain and loss in its cores. In addition to the linear coupling, the cores are also coupled through a cross-phase-modulation term. The authors demonstrate

that the focusing cross-phase-modulation interaction results in partial stabilization of the background wave, together with the Peregrine soliton. The stability region is found for PT-symmetric and antisymmetric bright solitons, with the latter presented in analytical form.

Kedziora *et al* [123] investigate the phase profiles of rogue wave solutions of the nonlinear Schrödinger equation. The authors focus specifically on the second-order rogue wave, in various forms, and extrapolate the results for higher-order structures. They show that the phase profile for any structure in the rogue wave hierarchy can be determined by examining phase bifurcations marked by zero-amplitude troughs.

Yan and Dai [124] consider self-similar optical rogue wave solutions and interactions for the generalized higher-order nonlinear Schrödinger (HONLS) equation with space- and time-modulated parameters. He presents a similarity transformation to reduce the generalized HONLS equation to the higher-order integrable Hirota equation with constant coefficients. In this way, he relates exact solutions of the generalized HONLS equation to the solutions of the integrable higher-order Hirota equation. He is able to generate self-similar rogue wave solutions of the HONLS equation by using the two lowest-order rational solutions of the higher-order Hirota equation.

As we can see from this short introduction, work on rogue waves is a dynamic area of research. This special issue mainly concentrates on optics. Nevertheless, it gives the flavour of the present state of the art on the subject of 'rogue waves'. As such, it is hoped that it will accelerate further studies in this important area of research.

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