Laser and Particle Beams (1987), *vol.* 5, *part* 1, *pp.* 3–14
Printed in Northern Ireland

On self-focusing of laser beams in plasma

By A. M. RUBENCHIK AND S. K. TURITSYN
Institute of Automation and Electrometry, Novosibirsk, USSR, 630090

(Received 26 November 1985)

Self-focusing of laser beams in plasma is studied analytically using the thermal (collision produced) and ponderomotive (nonlinear force produced) effects based on the model of Zakharov with solutions of the nonlinear Schrödinger equation. A basic difference appears if the electric field amplitude E is less than the threshold E_{th} (at which the electrodynamic energy density is equal to the gasdynamic pressure) from the contrary case, $E > E_{th}$. The length of the filamentation process is evaluated and results in large values for E below E_{th}.

1. Introduction

The self-focusing of electromagnetic radiation in the laser plasma leads to the appearance of a transverse inhomogeneity of density, temperature and local flow of the radiation. As a result the (plane) compression symmetry breaks, and a Rayleigh–Taylor instability of the target will be driven. The spatial inhomogeneity of the density and of other plasma parameters is well known from experiments even for moderate intensity of laser radiation (see, for example, Loipouch *et al.* [1]). The observed results change essentially from experiment to experiment; it is difficult to distinguish the range of occurrences directly connected with the self-focusing. Therefore the theoretical investigation of this processes seems important, to enable us to interpret the experiments.

Currently the modulation instability of the plane wave front has been well studied (Bespalov *et al.* 1966; Litvak *et al.* 1975). As a result of its development beam filamentation takes place. By the interpretation of experimental results it is natural to suppose that, depending on the length $\sim c/\gamma_{max}$, a stationary structure with dimensions of filaments $\sim K_{max}^{-1}$ is formed. Here, γ_{max} is the maximum increment of the modulation instability and K_{max} the wave number of the corresponding perturbations. However, the existence and stability of the self-focused waveguides, localized in the transverse direction, in which the nonlinear interaction compensates by diffraction spreading, is not obvious. Moreover, the characteristic time for observation in laser experiments greatly exceeds the time of the instability development. Therefore, the stationary nonhomogeneous structure can evidently be seen only if the filaments formed are stable formations.

This paper is devoted to the investigation of the conditions of the existence and stability of the localized filaments. The stationary self-focusing caused by ponderomotive forces (Hora 1969) has been studied in more detail (Max 1981; Vlasov *et al.* 1978). In the laser plasma, especially in experiments with shortwave radiation, the thermal mechanism of self-focusing plays an essential role (Sodha *et al.* 1976). In this case the possibility of the existence of stationary solutions, as well as that of their stability, is a problem.

In the first section we shall obtain equations describing the self-focusing with the simultaneous calculation of the nonlinear (ponderomotive) forces and thermal in-

0263-0364/87/0501-0003$05.00

stabilities. In the second section we shall show that at any correlation between the parameters of plasma there exist the stationary self-consistent distributions of the electric field and variations of the density and temperature, localized in the transverse direction. The properties of these solutions are investigated.

The problem of stability of the stationary solutions is naturaly divided into two parts. At first it is necessary to elucidate the stability of the solutions, in the limits of the stationary equations, relative to the variations of the amplitude and the distribution of the field at the entrance to the medium. It is known that all stationary distributions are unstable in the case of nonlinear-force self-focusing. We will show that for thermal self-focusing the filaments are absolutely stable in terms of the stationary equations. Moreover it is necessary to investigate the stability of the solutions relative to nonstationary perturbations. The third section is devoted to these questions. We will show that all stationary solutions are unstable. The instability is convective. The length $l_{ns} \sim c/\gamma$ over which it is developed is always much larger than the scale l_{st} on which breaking of the radiation into the filaments takes place in the frame of the stationary problem. Therefore, when the dimensions of the plasma L is such that $l_{ns} > L > l_{st}$, it is possible to observe stationary filamentation in the plasma. In concluding, on the basis of the obtained results, we will discuss the experimental situation.

2. The basic equations

Let us consider the propagation of the quasimonochromatic wave in isotropic, nonisothermal plasma. We will consider the case where characteristic times and scales of the field modulation are much larger than the period and wavelength of the laser light. By the usual method it is easy to obtain the equation for the envelope of the electric field ψ (Zakharov 1974)

$$i(\psi_t + V_g\psi_z) + \frac{V_g}{2k_0}\Delta\psi + \tfrac{1}{2}\omega''\psi_{zz} = \frac{\partial\omega}{\partial n}n\psi \tag{1}$$

Here,

$$V_g = \frac{c^2k}{\omega_p} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

is the group velocity of the electromagnetic wave, $\omega'' = d^2\omega/dz^2$, z axis is directed along the wave propagation, n is the perturbation of the plasma density under the action of nonlinear (ponderomotive) forces and the heating of the plasma.

The change of the local plasma temperature is described by the equation (Braginsky 1963)

$$n_0\frac{\partial T}{\partial t} = \varkappa\Delta_\perp T - \delta^2\nu_{ei}n_0T + \nu_{ei}\frac{|\psi|^2}{8\pi}\frac{\omega_p^2}{\omega^2} \tag{2}$$

Here, $\varkappa$ thermal conductivity, generally speaking, can be less than the classical value, $\varkappa = 3(nT/m)\nu_{ei}^{-1}$. The second term on the right side of (2) describes the heating of the plasma due to collision damping of the electromagnetic wave. The third term describes the loss of energy due to transfer to ions, radiative cooling and so on. Usually this term is too small, $\delta^2 \ll 1$, but its calculation is necessary to establish the steady state. As follows from (2), in such a condition,

$$\frac{\omega_p^2}{\omega^2}\int\frac{|\psi|^2}{8\pi}dr = \delta^2\int n_0T\,dr \tag{3}$$

Note that, because of the large group velocity of the laser radiation, it is enough to take thermal conductivity into account only in the transverse direction.

The plasma motion caused by nonlinear forces and heating due to radiation absorption can, as usual, be described in a linear approximation.

$$\begin{aligned}&\frac{\partial n}{\partial t}+\operatorname{div} n v_0=0\\&\frac{\partial v}{\partial t}+\frac{1}{Mn_0}\nabla(n_0T+T_0n)=-\frac{1}{n_0}\nabla\frac{|\psi|^2}{16\pi M}\end{aligned}\tag{4}$$

The term on the right hand side of the equation (4) (Miller's force or nonlinear force) describes the plasma repulsion under the action of ponderomotive interaction. Assuming, that the initial distribution of the density is homogeneous, and excluding it from the system (4), we obtain:

$$\frac{\partial^2 n}{\partial t^2}-\frac{T_0}{M}\Delta n=\frac{n_0}{M}\Delta T+\frac{1}{16\pi M}\Delta|\psi|^2,\ c_s^2=\frac{T_0}{M}\tag{5}$$

Let us notice that only the derivatives across the radiation distribution can be left here. Hence, it follows in particular that, the plasma motion does not influence the process of the self-focusing.

Let us write the system of equations in (1), (3) and (5) in the nondimensional form, introducing the variables

$$\begin{aligned}&t=\frac{k_0}{v_g}\frac{\varkappa}{n_0\nu_{ei}}\frac{\omega^2}{\omega_p^2}t'\\&r_\perp^2=\frac{\varkappa}{2n_0\nu_{ei}}\frac{\omega^2}{\omega_p^2}r_\perp'^2\quad T=\frac{v_gT_0\nu_{ei}}{k_0\varkappa}\frac{1}{\partial\omega/\partial n}\frac{\omega_p^2}{\omega^2}T'\\&z=\frac{k_0\varkappa}{n_0\nu_{ei}}\frac{\omega^2}{\omega_p^2}z'\quad|\psi|^2=16\pi\frac{v_gT_0n_0\nu_{ei}}{k_0\varkappa}\frac{\omega_p^2}{\omega^2}\frac{1}{\partial\omega/\partial n}|\psi'|^2\\&n=\frac{v_gn_0\nu_{ei}}{k_0\varkappa}\frac{1}{\partial\omega/\partial n}\frac{\omega_p^2}{\omega^2}n'\end{aligned}\tag{6}$$

As a result the system of equations acquires the form:

$$\begin{aligned}&i(\psi_t+\psi_z)+\Delta_\perp\psi+\alpha_3\psi_{zz}=n\psi\\&\alpha_1\frac{\partial T}{\partial t}=\Delta_\perp T-\eta^2T+|\psi|^2\\&\alpha_2n_{tt}-\Delta_\perp(n+T)=\Delta_\perp|\psi|^2\end{aligned}\tag{7}$$

where

$$\begin{aligned}&\eta^2=\frac{\omega^2}{2\omega_p^2}\delta^2,\quad\alpha_1=\frac{n_0v_g}{2\varkappa k_0}\\&\alpha_3=\frac{\omega''}{2}\frac{n_0\nu_{ei}}{\varkappa k_0^2}\frac{\omega_p^2}{\omega^2},\quad\alpha_2=\frac{\omega_p^2}{\omega^2}\frac{M_0n_0\nu_{ei}}{2T_0\varkappa}\frac{v_g^2}{k_0^2}\end{aligned}$$

So our problem is characterized by four nondimensional parameters. Here α_3 is always too small owing to the suggestion about the quasimonochromatic character of the wave and as will be shown below, the dispersion of the oscillations is practically always not essential in the problems under consideration. Since $\varkappa\sim n_0(v_T^2/\nu_{ei})$, the parameter $\alpha_1 \sim (c^2/v_T^2)(\nu_{ei}/\omega_p)$. As a rule this value is too small in laser plasma but it can be of order of unity for heavy targets.

The parameter η^2 is always too small, various processes contribute to it and its value is not clearly defined. However, as will be shown below, its value practically does not influence the properties of the solutions and their stability. As a rule, the parameter

$$\alpha_2 \sim \frac{c^4}{c_s^2 v_T^2}\left(\frac{\nu_{ei}}{\omega}\right)^2 \frac{\omega_p^2}{\omega^2}$$

is much greater than unity because of the great inertia of the ions.

3. Stationary filaments: Conditions for their existence

Consider stationary solutions (7), localized in the transverse direction. Neglecting the dispersion, we obtain:

$$i\psi_z + \Delta_\perp \psi - n\psi = 0 \tag{8}$$

$$\Delta_\perp T - \eta^2 T = -|\psi|^2 \tag{9}$$

$$n + T = -|\psi|^2 \tag{10}$$

Thus, the stationary solutions (8) to (10) are described by only one parameter—external parameter η^2. The properties of the solutions (8) to (10) are quite different for $n \gg 1$ and in the opposite limiting case. For $n \gg 1$ or for dimensional variables

$$\frac{n}{n_0} > \frac{c^2 n_0 \nu_{ei}}{\kappa \omega^2} \sim \left(\frac{c}{v_T}\frac{\nu_{ei}}{\omega}\right)^2 \tag{11}$$

we can neglect the density change because of the plasma heating. Here $n = -|\psi|^2$ and the criterion (10) becomes:

$$\frac{|\psi|^2}{8\pi n_0 T_0} > \frac{\nu_{ei} c^2 n_0}{\kappa \omega^2} \sim \left(\frac{c}{v_T}\frac{\nu_{ei}}{\omega}\right)^2 \tag{12}$$

Then, the system (8) to (10) is reduced to the nonlinear Schröndinger equation:

$$i\psi_z + \Delta_\perp \psi + |\psi|^2 \psi = 0 \tag{13}$$

Its properties are well studied. Equation (13) has a set of the localized stationary solutions—waveguides in which self-focusing compensates for a diffraction divergence (Yankauskas 1966). However all of them are unstable (Zakharov & Rubenchik 1974). The instability means that small deviations from the stationary distribution increase during the advance deep into plasma. Development of the instability leads to formation of local foci, in which the increase of the field is limited by nonlinear effects leading to the dissipation of the radiation energy.

The characteristic scale of the filamentation of the incident radiation and focus formation $l_\parallel \sim 1/|\psi|^2$ or in the dimensional variables is,

$$l_\parallel \sim \frac{V_g}{\omega_p} \frac{1}{\left(\dfrac{|\psi|^2}{8\pi n T}\right)} \tag{14}$$

The position of the foci changes strongly at small modifications of the intensity and profile of the laser radiation. Since the characteristic times of the observation are usually very large as compared with l/v_g, it is difficult to expect the observation of the stationary transverse filamentation of the radiation in this case.

In the opposite limiting case $n \ll 1$ or in the dimensional variables

$$\frac{n}{n_0} < \frac{\nu_{ei} c^2 n_0}{\varkappa \omega^2},$$

the nonlinear (ponderomotive) forces do exert essential influence on the deformation of the plasma profile. The density variation is conditioned by displacement of the plasma from the most heated regions. In this case the contribution of the nonlinear (ponderomotive) forces can be neglected in (9), $n = -T$ and the concentration distribution is defined by the constancy of the pressure in plasma. Self-focusing arises because of strong heating of plasma in the zone of increasing intensity and its corresponding displacement from the zone.

The equation describing the stationary self-focusing arrives at the form

$$\begin{aligned} &i\psi_z + \Delta_\perp \psi - n\psi = 0 \\ &\Delta_\perp n - \eta^2 n = |\psi|^2 \end{aligned} \tag{15}$$

As known (see e.g. Zakharov 1974) stable stationary localized solutions of the equation

$$i\psi_z + \Delta_\perp \psi + |\psi|^n \psi = 0$$

exist for $n < 2$.

Since $n \sim |\psi|^2 l_\perp^2$, and the transverse dimension of the waveguide decreases with the increase of $|\psi|^2$, n increases slower than $|\psi|^2$ and the system should have stable stationary waveguide solutions.

This result was obtained by Turitsyn (1985) strictly for the system of (15). Let us show that such solutions exist even in the limits of the more complete system (8) to (40), if the radiation intensity is less than some critical value. The system (8) to (10) is Hamiltonian and can be rewritten in the form: $i\psi_z = \delta H/\delta\psi^*$ with the additional conditions $\delta H/\delta\theta = 0$, $n + \theta = |\psi|^2$ where the Hamiltonian

$$H = \int \left[|\nabla\psi|^2 + \tfrac{1}{2}(\nabla\theta)^2 + \frac{\eta^2}{2}\theta^2 - \tfrac{1}{2}|\psi|^4 - \theta\,|\psi|^2 \right] dr \tag{16}$$

Besides H, (10) conserves the number of quanta $N = \int |\psi|^2\, dr$.

Let us consider the stationary solutions (8) to (10) of the form

$$\psi = \frac{e^{i\lambda^2 z}}{\sqrt{2}} f_0(r_\perp), \quad n = n_0(r_\perp), \quad \theta = \theta_0(r_\perp)$$

where f_0, n_0, θ_0 satisfy equations

$$\begin{aligned} &-\lambda^2 f_0 + \Delta_\perp f_0 = n_0 f_0 \\ &\Delta_\perp \theta_0 - \eta^2 \theta_0 = -\tfrac{1}{2} f_0^2 \\ &n_0 + \theta_0 = -\tfrac{1}{2} f_0^2 \end{aligned} \tag{17}$$

These solutions determine the structure of the stationary waveguides localized in the transverse direction. The system (16) can be obtained from the variational principle

$$\delta(H + \lambda^2 N) = 0 \tag{18}$$

In other words, the stationary solutions of (17) show an extremum of the Hamiltonian (16) for the fixed number of quanta and the additional condition $\delta H/\delta\theta = 0$.

Parameter λ^2 (nonlinear frequency shift) plays the role of the indefinite Lagrangian multiplier. Therefore it is sufficient to prove the boundedness of H from the following at fixed N for the proof of the existence of the solution.

Let us make use of the known inequality (Weinstein 1983):

$$\int |\psi|^4 \, dr_\perp \leqslant \frac{2}{N_{cr}} \int |\nabla_\perp \psi|^2 \, dr \int |\psi|^2 \, dr \tag{19}$$

where $N_{cr} \approx 11,68$ is the value $N[\psi]$ on the function ψ_0, which is the principle solution of the equation

$$-\psi_0 + \Delta_\perp \psi_0 + \psi_0^3 = 0$$

Let us estimate the integral $\int \theta \, |\psi|^2 \, dr$ with the help of the Hoelder inequality and (19)

$$\begin{aligned} \int \theta \, |\psi|^2 \, dr &\leqslant \left(\int \theta^4 \, dr\right)^{\frac{1}{4}} \left(\int |\psi|^4 \, dr\right)^{\frac{1}{4}} N^{\frac{1}{2}} \\ &\leqslant \left(\frac{2}{N_{cr}}\right)^{\frac{1}{2}} \left(\int (\nabla\theta)^2 \, dr\right)^{\frac{1}{2}} \left(\int \theta^2 \, dr\right)^{\frac{1}{4}} \left(\int |\nabla\psi|^2 \, dr\right)^{\frac{1}{4}} N^{\frac{3}{4}} \end{aligned}$$

Now we substitute this estimate into H and if the condition $N < N_{cr}$ is fulfilled, then:

$$\begin{aligned} H \geqslant & \left(1 - \frac{N}{N_{cr}}\right) \int |\nabla\psi|^2 \, dr + \frac{1}{2} \int (\nabla\theta)^2 \, dr + \frac{\eta^2}{2} \int \theta^2 \, dr \\ & - \left(\frac{2}{N_{cr}}\right)^{\frac{1}{2}} \left(\int (\nabla\theta)^2 \, dr\right)^{\frac{1}{2}} \left(\int \theta^2 \, dr\right)^{\frac{1}{4}} \left(\int |\nabla\psi|^2 \, dr\right)^{\frac{1}{4}} N^{\frac{3}{4}} \\ \geqslant & -\frac{N^3}{64 N_{cr}^2} \left(\frac{1+\eta^2}{\eta^2}\right)^2 \frac{1}{1 - \dfrac{N}{N_{cr}}} \end{aligned}$$

Thus, we have proved the boundedness of H when the radiation intensity N is less than N_{cr}. Stationary solutions exist also for $N > N_{cr}$. In this case they correspond to local extremum of H, which is is not bounded below. For understanding of the solution structure let us use the analogy of (17) with the equation for particle motion in the two-dimensional potential:

$$\begin{aligned} f_{0rr} + \frac{1}{r} f_{0r} &= -\frac{\partial}{\partial f_0} U(f_0, \theta_0) \\ \theta_{0rr} + \frac{1}{r} \theta_{0r} &= -\frac{\partial}{\partial \theta_0} U(f_0, \theta_0) \end{aligned} \tag{20}$$

$$U(x, y) = -\frac{\lambda^2}{2} x^2 - \frac{\eta^2}{2} y^2 + \tfrac{1}{2} y x^2 + \tfrac{1}{8} x^4 \tag{21}$$

Here r plays the role of time, and θ_0, f_0, the role of the particle coordinates. The potential form is represented in figure 1 ($\lambda = 1$).

Availability of the dissipative terms $(1/r)(\partial f_0/\partial r)$ and $(1/r)(\partial \theta_0/\partial r)$ (20) leads to the energy decrease. This eliminates the possibility of closed limiting cycles in the potential (21), and only limiting points $\theta_0 = f_0 = 0$ and $\theta_0 = 0$, $f_0 = \pm\sqrt{2}$ can serve as the

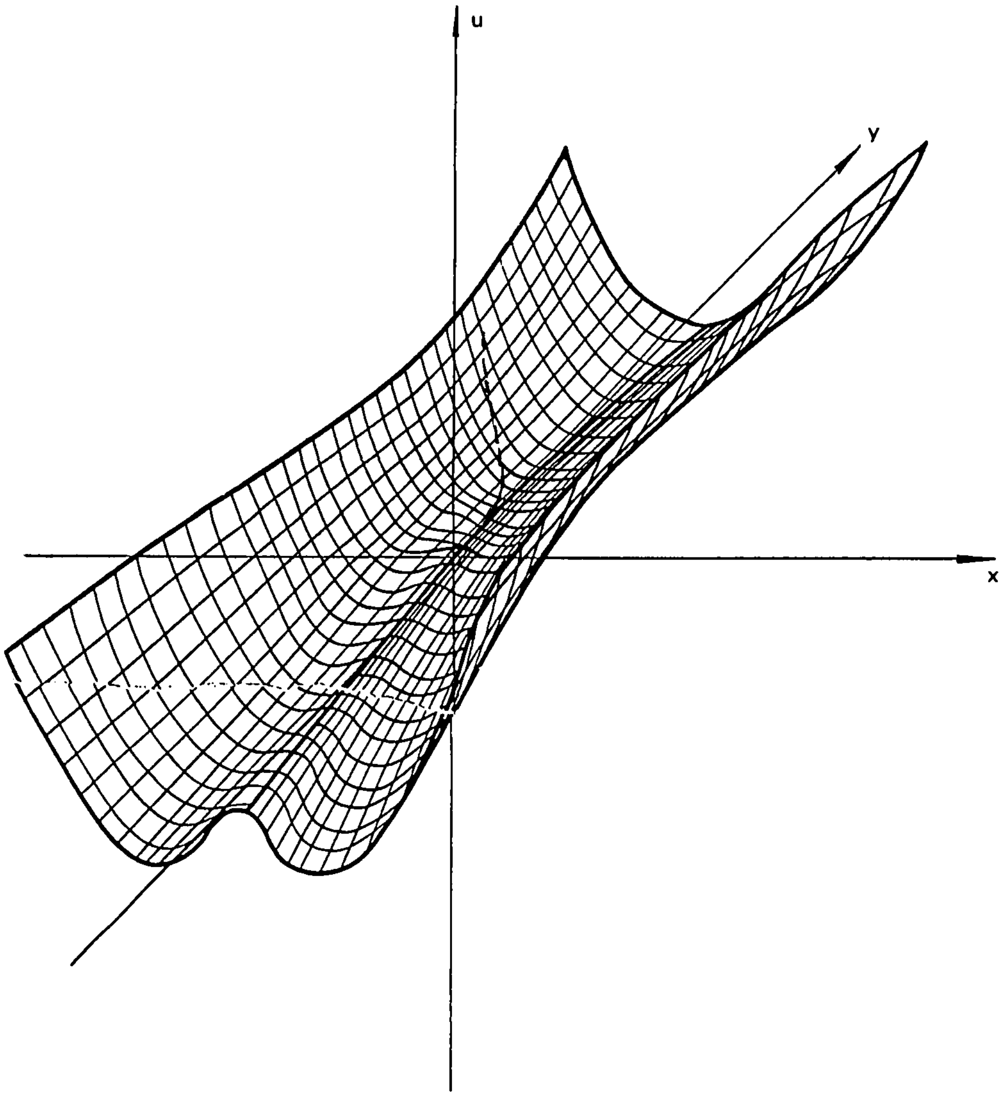

FIGURE 1. The effective potential (21). The trajectory presented on the picture corresponds to the basic most stable filament.

equilibrium positions. The trajectories ending in the points $\theta_0 = f_0 = 0$ correspond to the localized solutions. The trajectory corresponding to the basic waveguide (the waveguide with the minimum power) is represented in figure 1.

It is clear that it corresponds to the temperature perturbation $\theta > 0$. As η^2 is too small, the trajectory first goes quickly on the line $f_0 = 0$ during $\sim 1/\lambda$ and then slowly reaches the limiting point during the time $\sim 1/\eta$. It means, that the perturbation of temperature and density decrease monotonically from the maximum on the scale $\sim 1/\lambda$ so that the electric field drops practically to zero. The temperature distribution decreases exponentially, and this decrease is very slow because η is too small.

It is obvious, that there exist a great number of nonmonotonic solutions of (17) corresponding to trajectories, when the trajectory is multiply reflected from the walls before reaching the limiting point. These solutions correspond to the local extrema of

H; they are unstable and obviously are of no physical interest, as well as analogous solutions for the nonlinear Schnödinger equation.

As was discussed above, the filaments with a high energy field density are unstable. Boundedness of the Hamiltonian at $N<N_{cr}$ guarantees the stability of the basic waveguide according to the Lapunov theorem. This solution corresponds to the absolute minimum of the Hamiltonian and therefore any change of the profile increases H, but violates its preservation.

Thus we can suggest that for $\dfrac{|\psi|^2}{8\pi nT}<\dfrac{c^2 n_0 \nu_{ei}}{\varkappa\omega^2}$ the radiation on the scale l_{st} will break up into the stationary filaments.

If we substitute $\psi \sim e^{i\lambda^2 z}$ in to (15) it is easy to see that

$$l_\perp \sim \frac{1}{\lambda},\quad l_\parallel \sim \frac{1}{\lambda^2},\quad \frac{n}{n_0}\sim\lambda^2,\quad |\psi|^2\sim\lambda^4$$

If we introduce the field intensity in the center of a filament E_0^2 as a parameter, then from ratios (6) we have

$$\lambda^2 \sim \left(\frac{E_0^2}{8\pi nT}\right)^{\frac{1}{2}}\left(\frac{\omega^2\varkappa}{\nu_{ei}c^2 n_0}\right)^{\frac{1}{2}}$$

and, hence, decomposition into filaments occurs over the length

$$l_{st}=l_\parallel \sim \frac{k_0 c^2}{\omega_p^2}\frac{\varkappa\omega^2}{\nu_{ei}c^2 n_0}\frac{1}{\left(\dfrac{E_0^2}{8\pi nT}\right)^{\frac{1}{2}}\left(\dfrac{\varkappa\omega^2}{c^2\nu_{ei}n_0}\right)^{\frac{1}{2}}}$$

and characteristic transverse scale

$$l_\perp \sim \frac{c}{\omega_p}\left(\frac{\varkappa\omega^2}{n_0\nu_{ei}c^2}\right)^{\frac{1}{4}}\left(\frac{E_0^2}{8\pi nT}\right)^{-\frac{1}{4}}$$

4. Stability of the waveguides relative to nonstationary perturbations

Let us consider now the stability of the stationary solutions relative to perturbations of the form $\sim e^{-i\omega t+ikz}f(r_\perp)$. First we note that the perturbation velocity differs greatly from the group velocity because of the inertial character of the density and temperature modulation. That permits us not to take into account the dispersion of the electromagnetic wave later on. As was mentioned above, for $n\gg1$, when the self-focusing is conditioned by ponderomotive effects, the stationary filaments are unstable relative to small variations of the field distribution and, hence, the filaments do not occur. Therefore it is sufficient to limit oneself to consideration of the solutions with $n/n_0<\nu_{ei}n_0c^2/\varkappa\omega^2$, which are stable in terms of the stationary problem. Let us linearize (7) on the background of stationary solutions assuming:

$$\psi=e^{i\lambda^2 z}(f_0+f+ig),\quad T=\theta_0+\delta T,\quad n=n_0+\delta n.$$

For perturbations of the form $f, g, \delta T, \delta n \sim e^{ikz-i\omega t+im\varphi}$ we obtain the spectral problem:

$$\begin{aligned}
&(\omega-k)g+(\Delta_\perp-n_0-\lambda^2)f-f_0\delta n=0\\
&-(\omega-k)f+(\Delta_\perp-n_0-\lambda^2)g=0\\
&(-i\alpha_1\omega-\Delta_\perp+\delta^2)\delta T=2f_0 f\\
&-(\alpha_2\omega^2+\Delta_\perp)\delta n=\Delta_\perp\delta T
\end{aligned}\tag{22}$$

In general, the solution of the problem (22) on the eigenvalues is impossible, but the dispersion equation $\omega(K)$ can be obtained in the long wave limit $K \to 0$, using the method developed in by Zakharos *et al.* (1979). The idea of this method is the following. For $K = 0$ there exist marginal stable solutions of (22) corresponding to the derivatives of the stationary solution (7) with respect to the parameters. Let us consider perturbations locally close to indifferent stable modes at small K and define $\omega(K)$ using perturbation theory.

The increment of instability $\gamma = \mathrm{Im}\,\omega(K)$ represents the function of the wave vector K of the perturbations. We define the main peculiarities of $\gamma(K)$ behaviour. In the interval $\omega \ll \alpha_1/\alpha_2$ we neglect the member $\alpha_2\omega^2\delta n$ of the last equation of the system (22). Using this fact we turn to the equation:

$$(L_1 + \delta L)L_0 g = (\omega - k)^2 g. \tag{23}$$

Here the operators L_0, L_1, δL are

$$\begin{aligned} &L_0 = \Delta_\perp - n_0 - \lambda^2, \quad L_1 = L_0 + 2f_0 A_0^{-1} f_0, \\ &SL = 2f_0[A_\omega^{-1} - A_0^{-1}]f_0, \\ &A_\omega = -\Delta_\perp + \eta^2 - i\alpha_1\omega, \quad A_0 = -\Delta_\perp + \eta^2 \end{aligned} \tag{24}$$

Among the marginal stable modes one can separate the even modes along the transverse coordinates and odd ones which correspond to the displacement of the waveguide as a whole. From the result of Zakharov *et al.* (1974) it is clear that the odd modes have to be instable in the medium with the inertia of non-linearity. Indifferently, stable perturbations, i.e. solutions of the equation

$$L_1 L_0 g = 0, \tag{25}$$

are obtained by differentiation of the stationary solutions with respect to the transverse coordinate. In the cylindrical coordinate system it corresponds to the perturbations $\sim e^{i\varphi}$. Therefore, with $m = 1$, we obtain in the zeroth order,

$$L_1 L_0 g_0 = 0, \quad g_0 = f_{0r} \tag{26}$$

The zero eigenfunction of the conjugate problem is $g_0^* = f_{0r}$

$$L_0 L_1 g_0^* = 0$$

The spectrum $\omega(K)$ is obtained from (23) as the solvability condition for the first order k^2:

$$\omega = i\frac{2}{\alpha_1}\frac{\langle f_0 \mid f_0\rangle}{\left\langle \frac{\partial}{\partial r}\Delta_\perp^{-1} f_0^2 \,\middle|\, \frac{\partial}{\partial r}\Delta_\perp^{-1} f_0^2 \right\rangle}k^2 \tag{27}$$

The sign $\langle\ \rangle$ denotes integration with respect to the transverse coordinates, $\Delta_\perp^{-1}$ is the operator inverse to the two dimensional Laplace operator. The expression (27) is valid up to $\omega \approx \alpha_1/\alpha_2$ and $k \approx \lambda\alpha_1/\sqrt{\alpha_2}$. The following characteristic portion on the axis ω lies in the interval $\alpha_1/\alpha_2 \ll \omega \ll \lambda/\sqrt{\alpha_2}$. In this case the spectral problem has the form (25) as before where the operators A_ω, A_0 are

$$A_\omega = -(\Delta_\perp + \alpha_2\omega^2), \quad A_0 = -\Delta_\perp$$

Multiplying (25) by f_{0r} from the left hand side and substituting it, we obtain after transformations:

$$(\omega - k)^2 = -\frac{4}{N}\langle f_0 f_{0r}\,|A_\omega^{-1} - A_0^{-1}|\,f_0 f_{0r}\rangle = -\frac{4\alpha_2\omega^2}{N}C(\lambda) \tag{28}$$

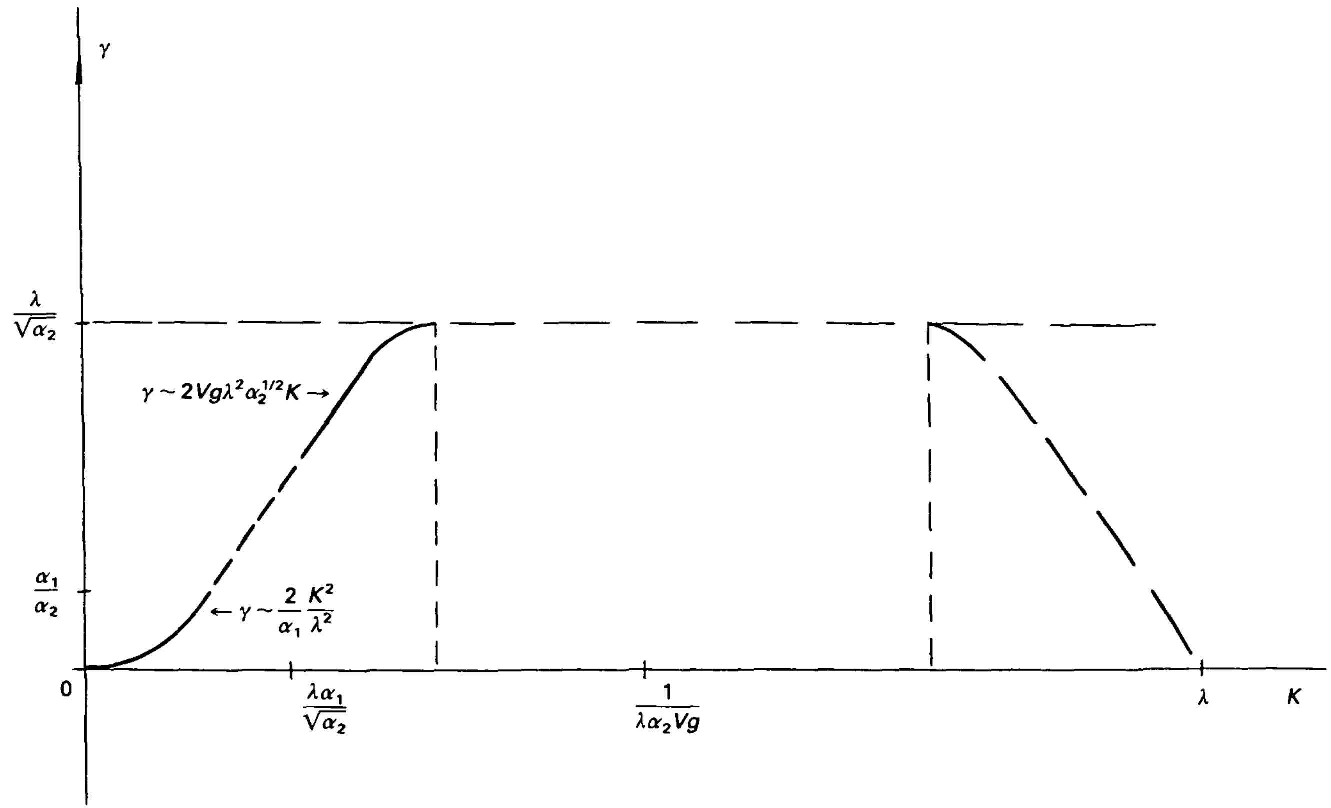

FIGURE 2. Growth rate of the filament instability as function of K. Maximum growth rate is

$$\gamma \sim \omega_p\left(\frac{E_0^2}{8\pi nT}\right)^{\frac{1}{4}}\frac{c_s}{\sqrt{v_T c}}$$

where $C(\lambda) = \langle f_0 f_{0r} |\Delta_{\perp}^{-2}| f_0 f_{0r}\rangle$ is the dimensionless structure factor. Equation (28) has the solution describing the unstable branch

$$\omega = K \frac{1 + i\dfrac{2\alpha_2^{\frac{1}{2}} C^{\frac{1}{2}}(\lambda)}{N^{\frac{1}{2}}}}{1 + \dfrac{4\alpha_2 C(\lambda)}{N}} \tag{29}$$

It is clear that in this region that the increment results in the linear increase by K. The expression for the increment (29) is valid up to $k \sim 1/\lambda\alpha_2$.

For large k, characteristic times of the density modulation increase become comparable with the acoustic ones. In this case it is necessary to take into account the inertia in the sound equation. With the help of the arguments analagous to those cited above, it can be shown that for $\omega > \lambda/\sqrt{\alpha_2}$, the instability is absent. It means that the instability increment remains approximately constant in the interval $1/\lambda\alpha_2 < K < 1/\lambda$. Instability stops when the perturbation wavelength becomes of the order of the transverse size of the waveguide λ. Figure 2 shows the dependence of the increment on the wave vector of perturbation.

It should be noted that as well as for modulation instability the maximum increment, $\gamma \sim \lambda/\sqrt{\alpha_2}$,

$$\left(\text{or in dimensional variables } \gamma \sim \omega_p \left(\frac{E_0^2}{8\pi n T}\right)^{\frac{1}{4}} \left(\frac{c_s^4}{v_T^2 c^2}\right)^{\frac{1}{4}}\right)$$

in the medium with pressure-nonlinearity is considerably less than the nonlinear frequency shift. The meaning of the obtained nonlinearity is evident immediately. As was mentioned above, the considered marginally stable mode corresponds to the shift of the waveguide as a whole. Hence, the dependence e^{ikz} denotes a waveguide being bent along the axis z on the scale $\sim 1/k$. In the stationary state nonlinearity compensates for diffraction divergence. In the case of waveguide shift due to inertia of the medium, nonlinearity decreases in comparison with a stationary value, which leads to nonlinearity. It is clear that nonlinearity leads to radiation splitting but not to the local increase of the field. Therefore, due to the instability, neither amplification of anomalous absorption nor radiation scattering should take place. The instability obtained is convective. For its development it is necessary for the plasma size to exceed the value $v_g/\gamma_{\max}$. Let us emphasize that this size is much larger than the length of the formation of the stationary waveguide $v_g/\Delta\omega_{NL}$. It should be also noted that such filamentation can only be observed at oblique incidence on the target. At normal incidence due to self-focusing of incident and reflected waves, the instability becomes absolute.

5. Conclusions

We have shown that self-focusing of laser beams in plasma differs greatly when E^2 is greater than E_{th}^2 from the inverse case. The value of

$$\frac{E_{th}^2}{8\pi n_0 T_0} \approx \frac{\nu_{ei} c^2 n_0}{\varkappa \omega^2} \sim \frac{c^2}{v_T^2}\left(\frac{\nu_{ei}}{\omega}\right)^2 \textit{ is } 10^{-2},$$

for a neodymium laser, a polyethylene target ($z \sim 5$), $n \sim n_c$ and a temperature $\sim$1 Kev. Therefore for most experiments with moderate radiation intensities ejection of plasma under the action of the nonlinear (ponderomotive) forces plays a crucial role for self-focusing. Since $E_{th}^2 \sim n^2 z^2 \sim \omega^4 z^2$ the thermal mechanism of self-focusing is essential for the short-wave radiation and targets of heavy materials.

For $E^2 > E_{th}^2$ self-focusing leads to the formation of local field maxima and can increase anomalous absorption of the radiation. Self-focusing may even lead to more homogeneous irradiation of the critical density surface. In fact, instability of self-focusing leads to focus motion in the transverse direction and, as a result, to average alignment of the target irradiation.

For $E^2 < E_{th}^2$ as was shown above, stationary filaments appear in the plasma. The length of instability development is

$$l \sim \frac{v_g}{\gamma_{\max}} \approx \frac{v_g}{\omega_p} \frac{c}{v_T} \sqrt{\frac{M}{m}} \left(\frac{E E_{th}}{8\pi n T}\right)^{-\frac{1}{2}}$$

For energy densities $E^2 \sim E_{th}^2$, and plasma densities of the order of the critical length of the instability, development exceeds several thousand wavelengths. Hence, in real experiments where the plasma size is small, stationary filaments may be formed. Such filaments are well observed in experiments (Limpouch *et al.* 1984). A value $E^2/E_{th}^2 \leqslant 0,2$ is obtained for them, i.e. their existence is in good agreement with the results of this paper. Let us emphasize once more that one can expect filament formation only in the case of oblique incidence of radiation. Stretched along the light incidence directions, the filaments differ by nature from "the jets" with normal orientation to the critical density surface, the formation of which is conditioned by other mechanisms.

Filamentation of the radiation leads to increasing of anomalous processes; it can explain, for example, generation of the harmonic $\frac{3}{2}\omega$ in the experiments with the 3rd harmonic of the neodimium laser. In these experiments at a uniform energy distribution along the spot, the collision threshold is hardly exceeded due to high plasma density and due to collisions. Parametric instability can arise due to a local increase of the field energy induced by self-focusing.

REFERENCES

Bespalov, V. I. & Talanov, V. I. 1966 *Pis'ma JETP,* **3,** 471.

Braginsky, S. I. 1963 *Voprosy teoryi plasmy (Atomisdat, Moscow),* **1,** 183.

Hora, H. 1969 *Z. Phys.* **226,** 156.

Limpouch, I. & Rozanov, V. B. 1984 *Kvant, Elektr.* **11,** 1415.

Litvak, A. G., Mironov, V. A., Frayman, G. M. & Unakovsky, A. D. 1975 *Fisica Plasmy,* **1,** 6071.

Max, C. E. 1981 *Lawrence Livermore National Laboratory,* UCRL-53107.

Sodha, M. S., Chatak, A. K. & Tripathi, V. K. 1976 *Progress in Optics,* E. Wolf, ed. (Acad. Press, New York) Vol. 13, p. 171.

Turitsyn, S. K. 1985 *Teor. Math. Phys.* **64,** 2.

Vlasov, S. N., Piskunova, L. V. & Talanov, V. I. 1978 *JETP,* **75,** 5, 1602.

Weinstein, M. I. 1983 *Comm. Math. Phys.* **87,** 567.

Yankauskas, V. 1966 *Izv. Vuzov, Radiofizika,* **9,** 523.

Zakharov, V. E. 1974 *Izv. Vuzov, Radiofizika,* **17,** 43.

Zakharov, V. E. & Rubenchik, A. M. 1974 *Sov. Phys. JETP,* **38,** 494.